# Gauge protected entanglement between gravity and matter

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# TOPICS

- **Introduction**
- **Entanglement in the canonical QG**
- **Conclusions and outlook**

# INTRODUCTION

**Important foundational notions in quantum theory:**

- entanglement,
- decoherence,
- measurement.

**Important foundational notions in general relativity:**

- principle of general relativity: diffeomorphism invariance and background independence,
- equivalence principle.

**Combining these principles in a full QG theory may lead to unexpected consequences.**

***In particular, relationship between gauge symmetry and entanglement!***

# INTRODUCTION

**Statement in brief:**

- couple matter to gravity (minimal coupling, equivalence principle),
- perform the quantization of the gravity-matter system (or imagine someone else did),
- look for physical, gauge-invariant states (diff-invariance, principle of general relativity),
- for all such states, **gravity and matter are entangled!**

**Symbolically, for $\mathcal{H} = \mathcal{H}_G \otimes \mathcal{H}_M$,**

$$|\Psi_{\text{physical}}\rangle = c_1|g_1\rangle \otimes |\phi_1\rangle + c_2|g_2\rangle \otimes |\phi_2\rangle + \ldots$$

**Formally, the gauge-invariant subspace $\mathcal{H}_{\text{phys}}$ of the total Hilbert space $\mathcal{H}$ contains no separable states:**

$$(\forall|\Psi\rangle \in \mathcal{H}_{\text{phys}}) \qquad |\Psi\rangle \neq |g\rangle \otimes |\phi\rangle$$

**(except maybe by accident).**

# ENTANGLEMENT IN CANONICAL QG

**Setup and the classical theory:**

- Start from some action for gravity and matter,

$$S[g, \phi] = S_G[g] + S_M[g, \phi] \,,$$

- introduce momenta for fundamental variables $g$ and $\phi$,

$$\pi_g \equiv \frac{\delta S}{\delta \partial_0 g} \,, \qquad \pi_\phi \equiv \frac{\delta S}{\delta \partial_0 \phi} \,,$$

- perform the Dirac analysis for constrained systems to find the Hamiltonian in the form

$$H = \int_{\Sigma_3} d^3\vec{x} \left[ N\mathcal{C} + N^i \mathcal{C}_i + N^{ab} \mathcal{C}_{ab} \right] ,$$

- where the constraints are the ten generators of the local Poincaré symmetry, in the form:

$$\begin{aligned}
\mathcal{C} &= \mathcal{C}^G(g, \pi_g) + \mathcal{C}^M(g, \pi_g, \phi, \pi_\phi) \,, \\
\mathcal{C}_i &= \mathcal{C}_i^G(g, \pi_g) + \mathcal{C}_i^M(g, \pi_g, \phi, \pi_\phi) \,, \\
\mathcal{C}_{ab} &= \mathcal{C}_{ab}^G(g, \pi_g) + \mathcal{C}_{ab}^M(g, \pi_g, \phi, \pi_\phi) \,.
\end{aligned}$$

# ENTANGLEMENT IN CANONICAL QG

**Canonical quantization (Heisenberg picture):**

- promote gravitational and matter fields to operators,

$$g \to \hat{g}\,, \qquad \pi_g \to \hat{\pi}_g\,,$$
$$\phi \to \hat{\phi}\,, \qquad \pi_\phi \to \hat{\pi}_\phi\,,$$

- promote Dirac brackets to commutators,

$$\{\,\cdot\,,\,\cdot\,\}_D \to [\,\cdot\,,\,\cdot\,]\,,$$

- impose Gupta-Bleuler-like conditions for the state vectors:

$$\hat{\mathcal{C}}|\Psi\rangle = 0\,, \qquad \hat{\mathcal{C}}_i|\Psi\rangle = 0\,, \qquad \hat{\mathcal{C}}_{ab}|\Psi\rangle = 0\,.$$

**Make sure everything is well defined, unique, etc. . .**

# ENTANGLEMENT IN CANONICAL QG

**Study the structure of the constraint equations:**

- the matter-parts of the 3-diffeo and local Lorentz constraints have the benign form

$$\mathcal{C}_i^M = \pi_\phi \nabla_i \phi \,, \qquad \mathcal{C}_{ab}^M = \pi_\phi M_{ab} \phi \,,$$

- while the matter-part of the scalar constraint features the matter Lagrangian:

$$\mathcal{C}^M = \pi_\phi \nabla_\perp \phi - \frac{1}{N} \mathcal{L}_M(g, \pi_g, \phi, \pi_\phi) \,.$$

- Choose some nice representation,

$$\langle g | \hat{g} = g \langle g | \,, \qquad \langle g | \hat{\pi}_g = -i \frac{\delta}{\delta g} \langle g | \,, \qquad \langle \phi | \hat{\phi} = \phi \langle \phi | \,, \qquad \langle \phi | \hat{\pi}_\phi = -i \frac{\delta}{\delta \phi} \langle \phi | \,,$$

- and rewrite the scalar constraint as a functional differential equation:

$$\left[ \mathcal{C}^G(g, \tfrac{\partial}{\partial g}) + \mathcal{C}^M(g, \tfrac{\partial}{\partial g}, \phi, \tfrac{\partial}{\partial \phi}) \right] \Psi[g, \phi] = 0 \,.$$

# ENTANGLEMENT IN CANONICAL QG

**Look for separable solutions:**

$$|\Psi\rangle = |\Psi_G\rangle \otimes |\Psi_M\rangle \qquad \Rightarrow \qquad \Psi[g,\phi] = \Psi_G[g]\Psi_M[\phi]\,.$$

**But the scalar constraint equation does not have any such solutions!!!**

**Namely, if the scalar constraint equation allows for the separation of variables, the matter-part must have the form**

$$\mathcal{C}^M(g,\tfrac{\partial}{\partial g},\phi,\tfrac{\partial}{\partial \phi}) = \mathcal{K}_G(g,\tfrac{\partial}{\partial g})\;\mathcal{K}_M(\phi,\tfrac{\partial}{\partial \phi})\,,$$

**but the inspection of Lagrangians shows that it <u>does not</u> have the required form:**

$$\mathcal{L}_M^{\text{scalar}} = \sqrt{-g}\left[g^{\mu\nu}(\partial_\mu\varphi)(\partial_\nu\varphi) - m^2\varphi^2 + U(\varphi)\right],$$

$$\mathcal{L}_M^{\text{Dirac}} = \sqrt{-g}\left[\frac{i}{2}\bar{\psi}\gamma^a e^\mu{}_a \nabla_\mu\psi - m\bar{\psi}\psi + \text{c.c.}\right],$$

$$\mathcal{L}_M^{\text{Yang-Mills}} = \sqrt{-g}\left[-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}\,\text{tr}\,F_{\mu\nu}F_{\rho\sigma}\right].$$

# ENTANGLEMENT IN CANONICAL QG

**Conclusion:**

***Separable states are not gauge invariant!!!***

**What about the other two constraints?**

- the local Lorentz constraint does admit separable states as solutions,

$$\mathcal{C}^M_{ab} = \mathcal{C}^M_{ab}(\phi, \pi_\phi) = \pi_\phi M_{ab} \phi \,,$$

- the 3-diffeo constraint admits separable states for the scalar field,

$$\mathcal{C}^M_i(\varphi, \pi_\varphi) = \pi_\varphi \partial_i \varphi \,,$$

  but not for fields of higher spin,

$$\mathcal{C}^M_i(\psi, \pi_\psi) = \pi_\psi \nabla_i \psi \,.$$

- Similarly for internal gauge symmetries such as $SU(3) \times SU(2) \times U(1) \ldots$

# CONCLUSIONS AND OUTLOOK

**Main result:**

***Matter and gravity are always entangled!***

**Consequences:**

- Matter is never in a pure state — after tracing out the gravitational degrees of freedom, the reduced density matrix for matter fields is never a pure state.
- Important for the study of decoherence — one usually starts from an initial separable state, which becomes entangled over time. But even the initial state cannot be separable.
- Gauge-protected entanglement leads to effective "exchange interaction" due to the overlap between states, like for the Pauli exclusion principle. This effective interaction gives rise to deviations from geodesic trajectories for particles, introducing small violation of the weak equivalence principle.
- Numerical calculations in toy-models suggest that the amount of entanglement is rather small, compatible with the semiclassical picture of spacetime.

***THANK YOU!***