ADS 4D/BPS 3D Correspondence

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with Csaba Csaki, Yuri Shirman

Outline

A Brief History of Monopoles

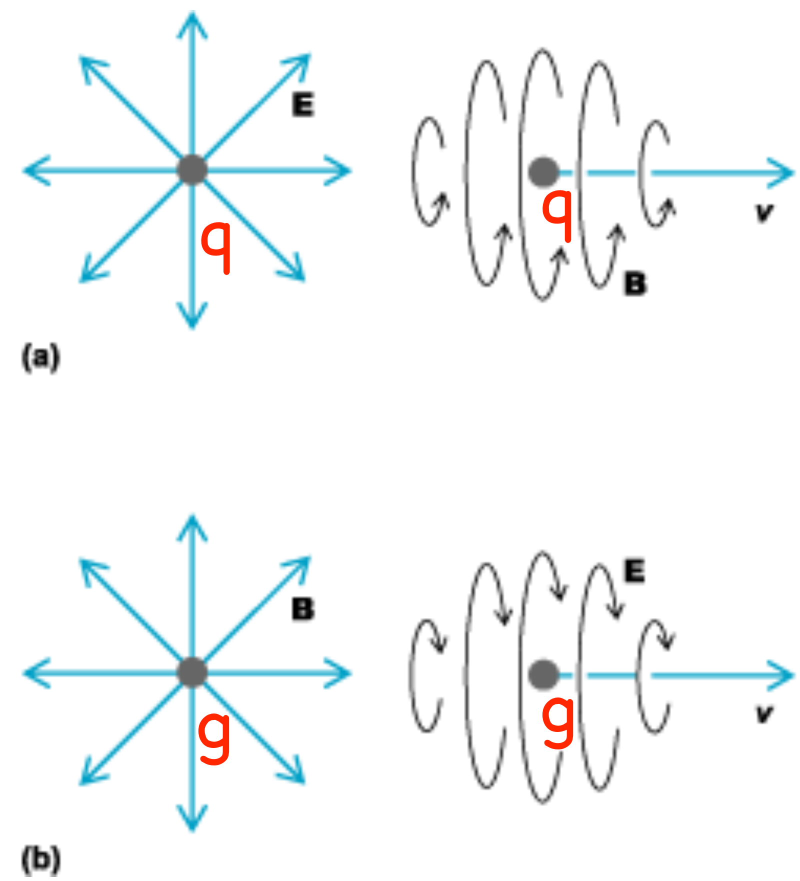
SUSY: 4D -> 3D x S^1

N=2 SUSY in 4D

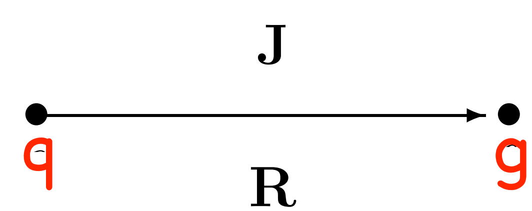
Standard Model

Conclusions

J.J. Thomson

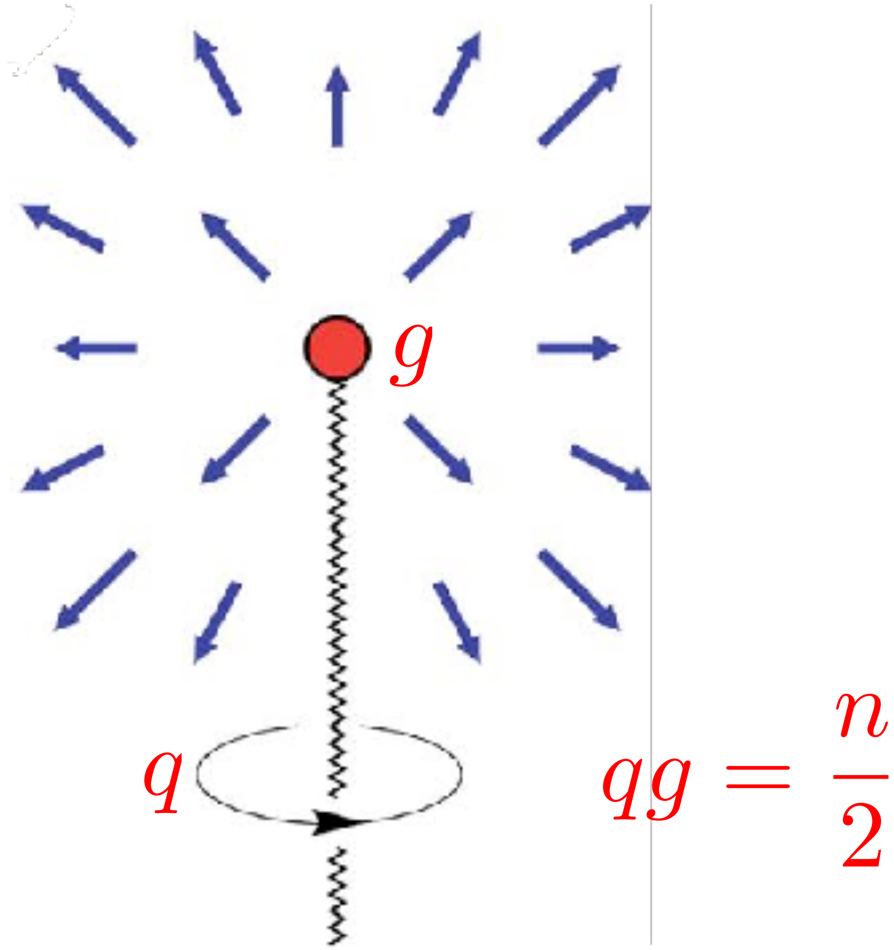


$$J = q\,g$$



Philos. Mag. 8 (1904) 331

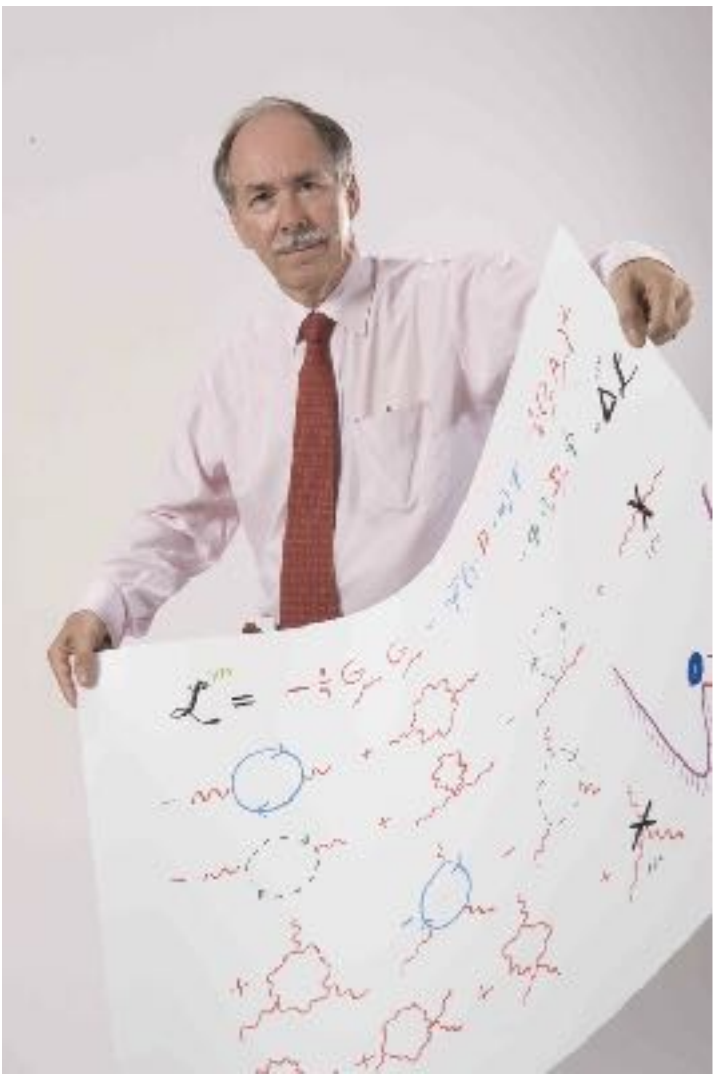
Dirac

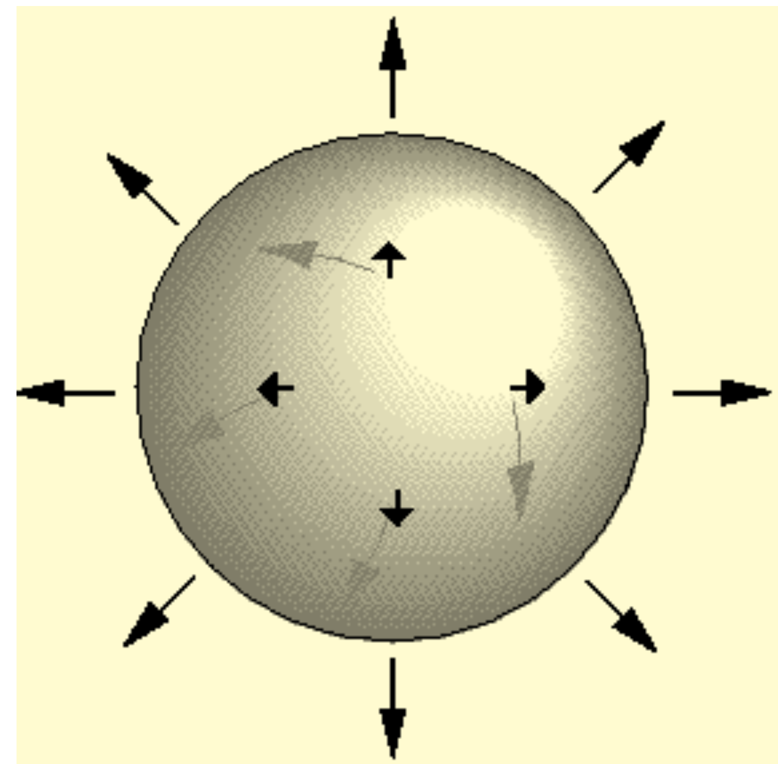


charge quantization

Proc. Roy. Soc. Lond. A133 (1931) 60

‘t Hooft-Polyakov

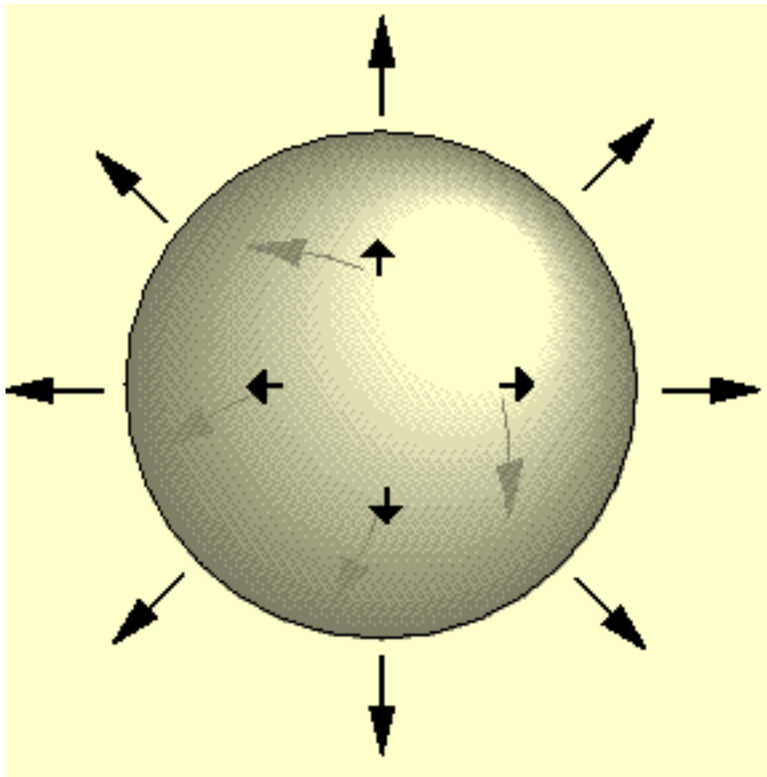




topological monopoles

Nucl. Phys., B79 (1974) 276
JETP Lett., 20 (1974) 194

`t Hooft–Polyakov

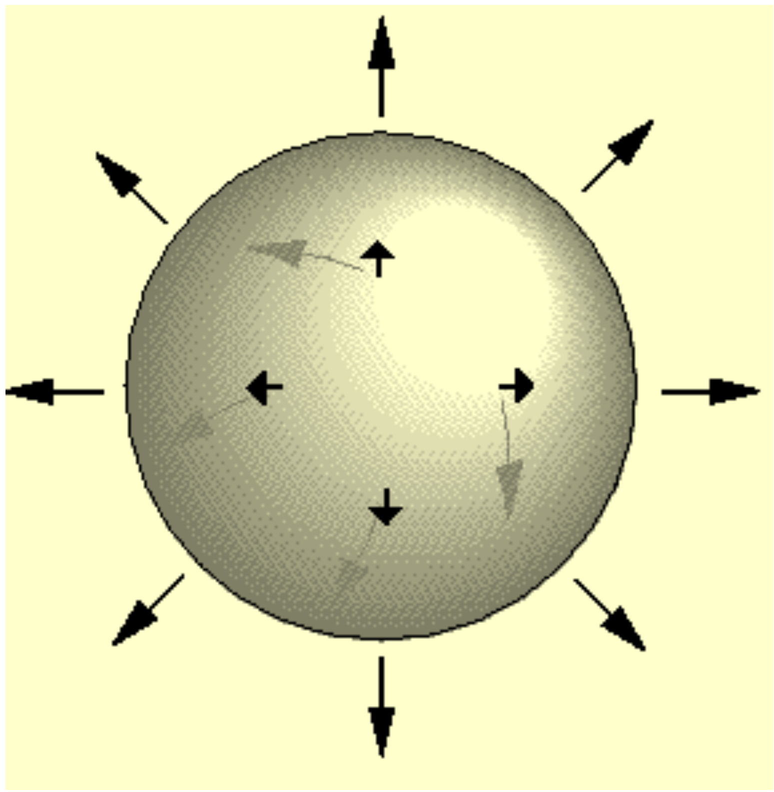


hedgehog gauge

$$\phi^a = \hat{r} v \, h(vr)$$

$$W_i^a = \epsilon^{air} \hat{r}^j \frac{f(vr)}{gr}$$

‘t Hooft-Polyakov



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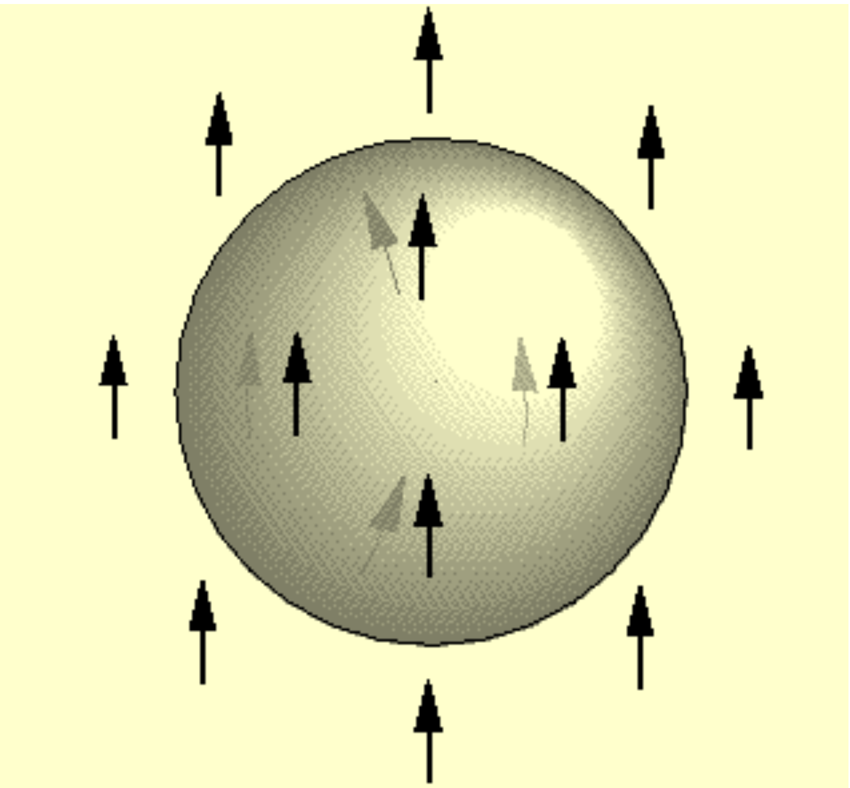
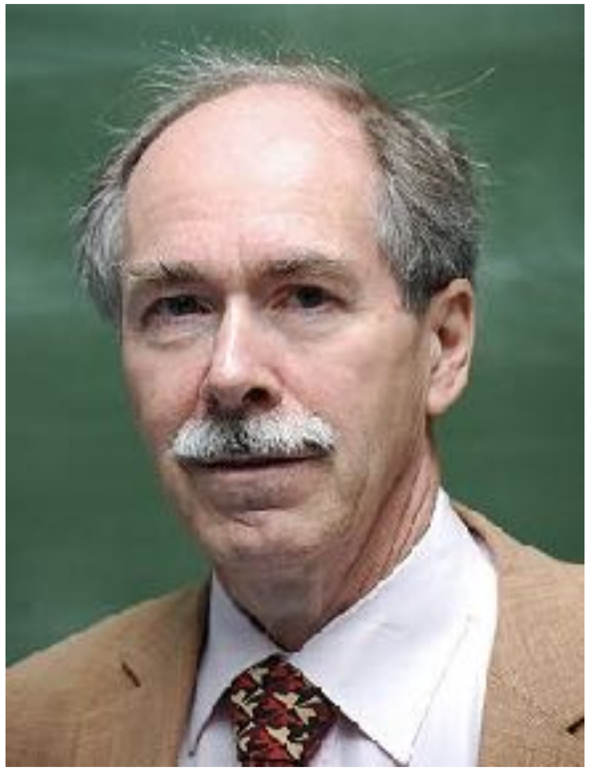


singular gauge

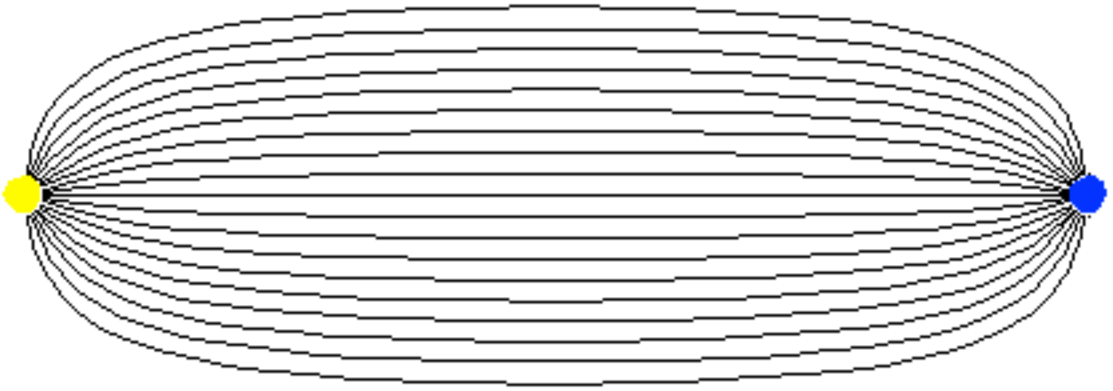
$$U^\dagger \tau^a \phi^a U = v\, h(vr) \tau^3$$

$$U = \frac{1}{\sqrt{2}} \left(\sqrt{1+\hat{r}_3} I + i\, \frac{\hat{r}_2 \sigma^1 - \hat{r}_1 \sigma^2}{\sqrt{1+\hat{r}_3}} \right)$$

`t Hooft-Mandelstam



magnetic condensate
confines electric charge



High Energy Physics Ed. Zichichi, (1976) 1225
Phys. Rept. 23 (1976) 245

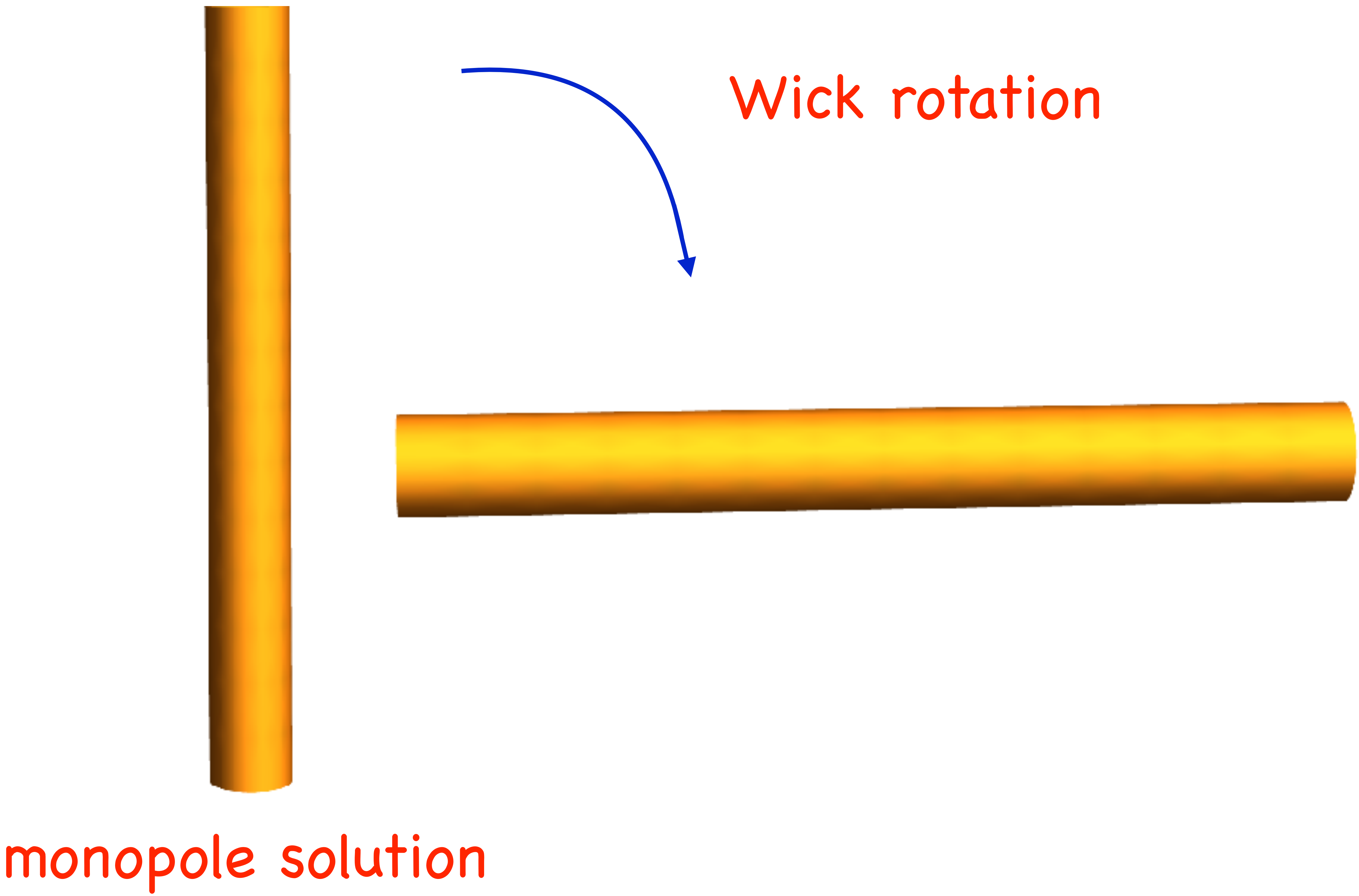
4D -> 3D x S^1

SUSY SU(N) with F flavors

$$W_\mu^a \to \vec{W}, \phi^a$$

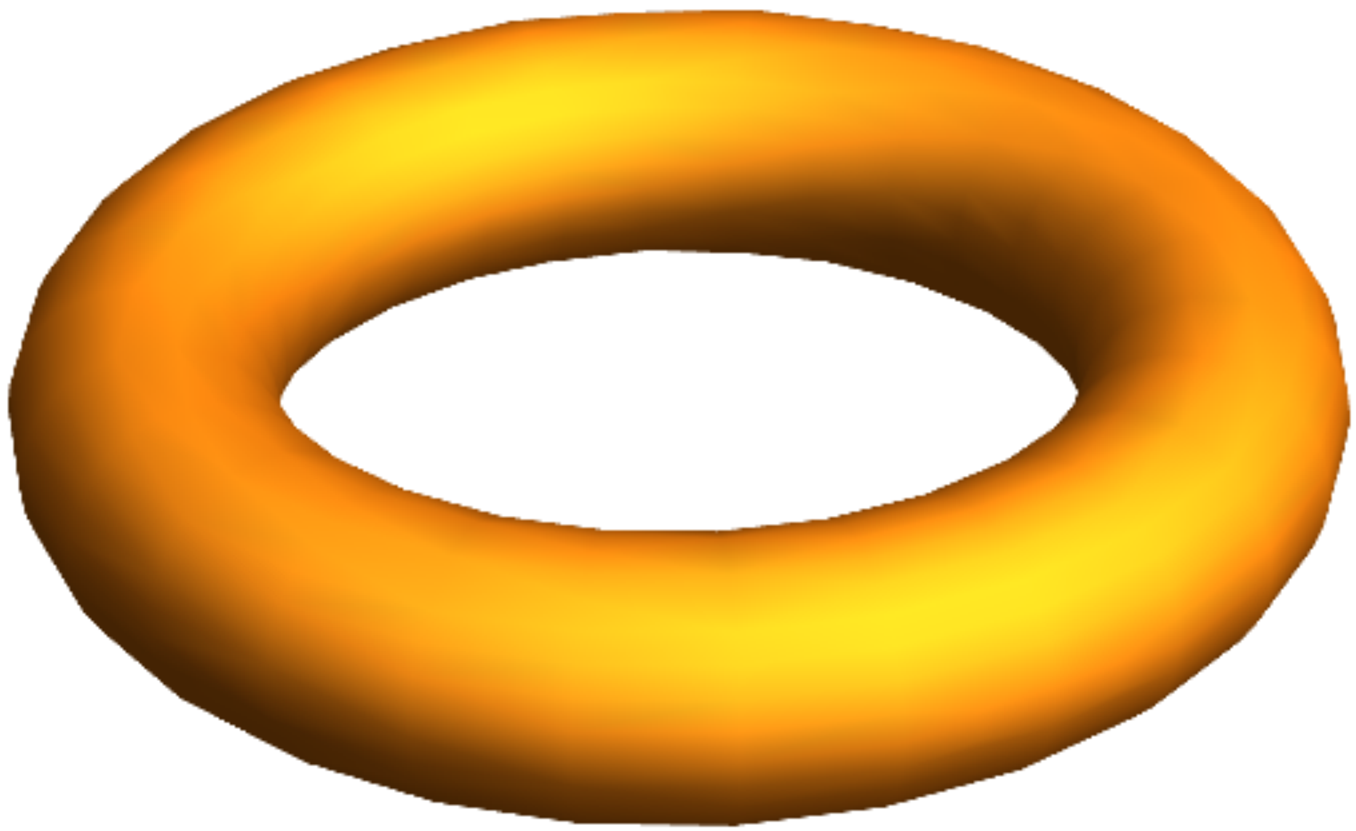
monopole solution

4D -> 3D x S^1



4D -> 3D x S^1

compactify

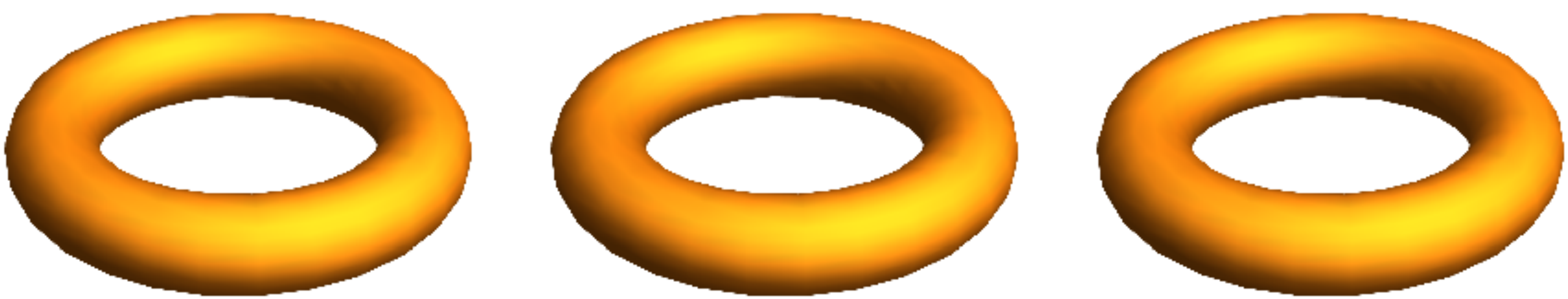


monopole solution

N-1 Embeddings of SU(2)

N-1 diagonal generators

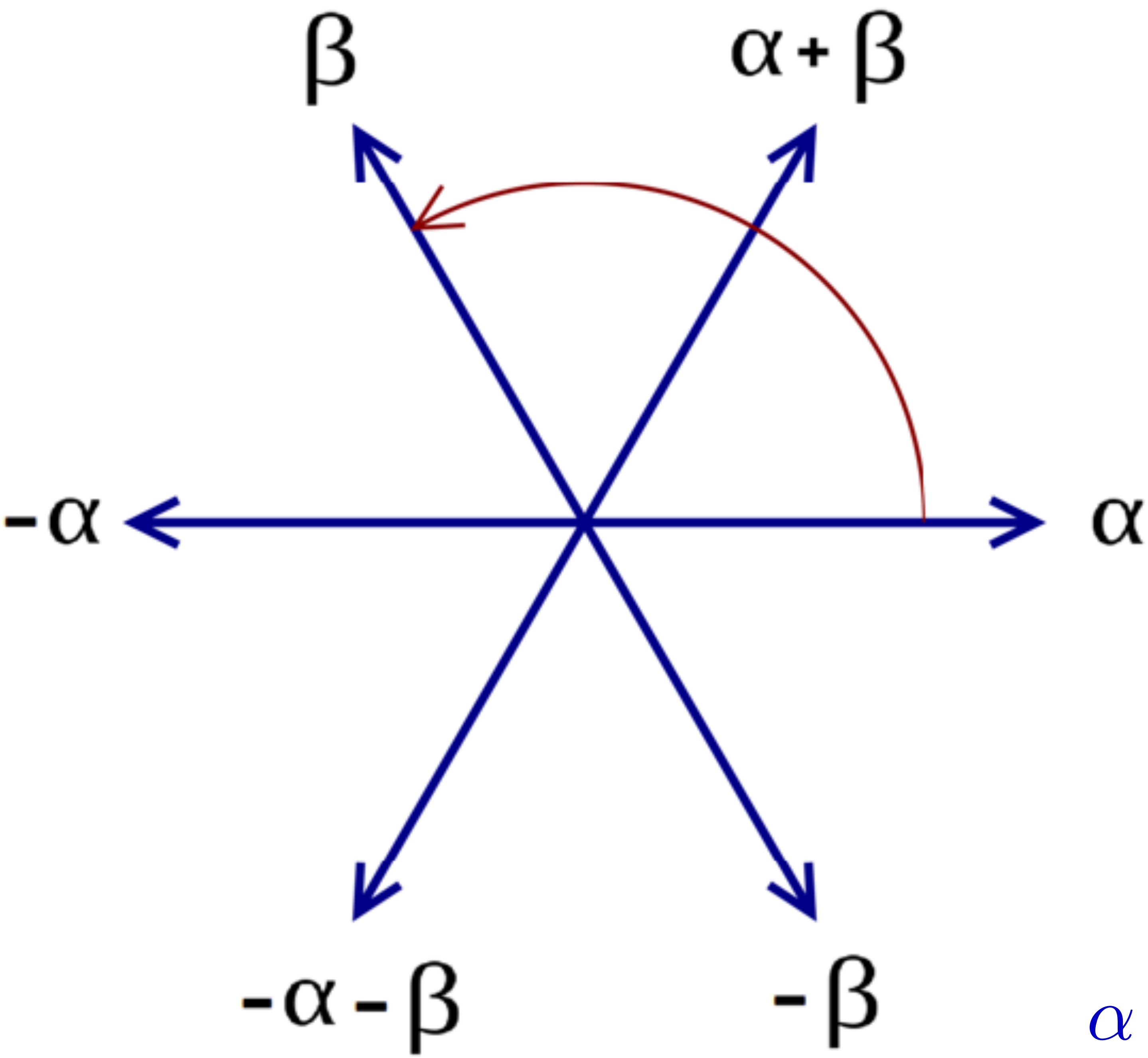
$$\begin{pmatrix} \frac{1}{2} & 0 & 0 & \dots \\ 0 & -\frac{1}{2} & 0 & \dots \\ 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & \dots \\ 0 & \frac{1}{2} & 0 & \dots \\ 0 & 0 & -\frac{1}{2} & \dots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ 0 & 0 & \frac{1}{2} & \dots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \quad \dots$$



$\dots$

monopole solutions

Roots of SU(3)



$$\mathbf{H} = (T^3, T^8)$$

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} = \alpha \cdot \mathbf{H}$$

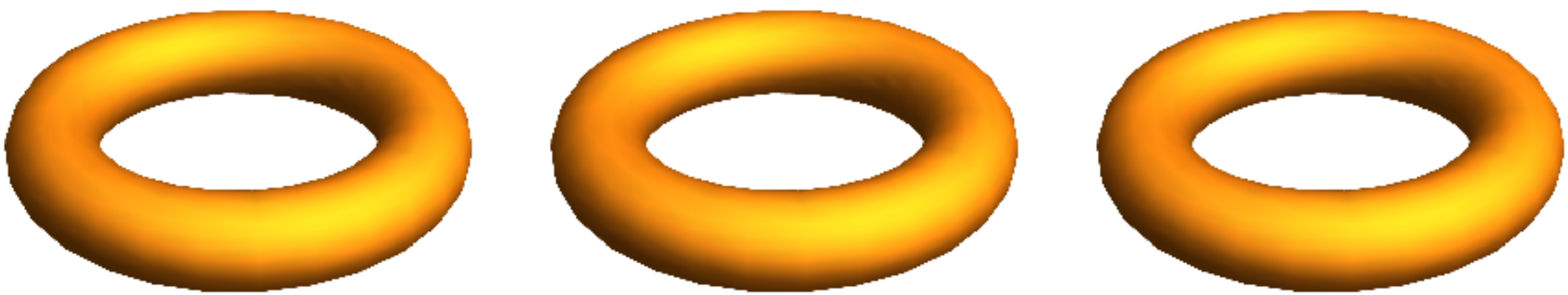
$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix} = \beta \cdot \mathbf{H}$$

$$\alpha = (1, 0) \qquad \beta = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$$

N-1 Embeddings of SU(2)

N-1 diagonal generators

$\alpha_1 \cdot \mathbf{H}$ $\alpha_2 \cdot \mathbf{H}$ $\alpha_3 \cdot \mathbf{H}$ $\cdots$

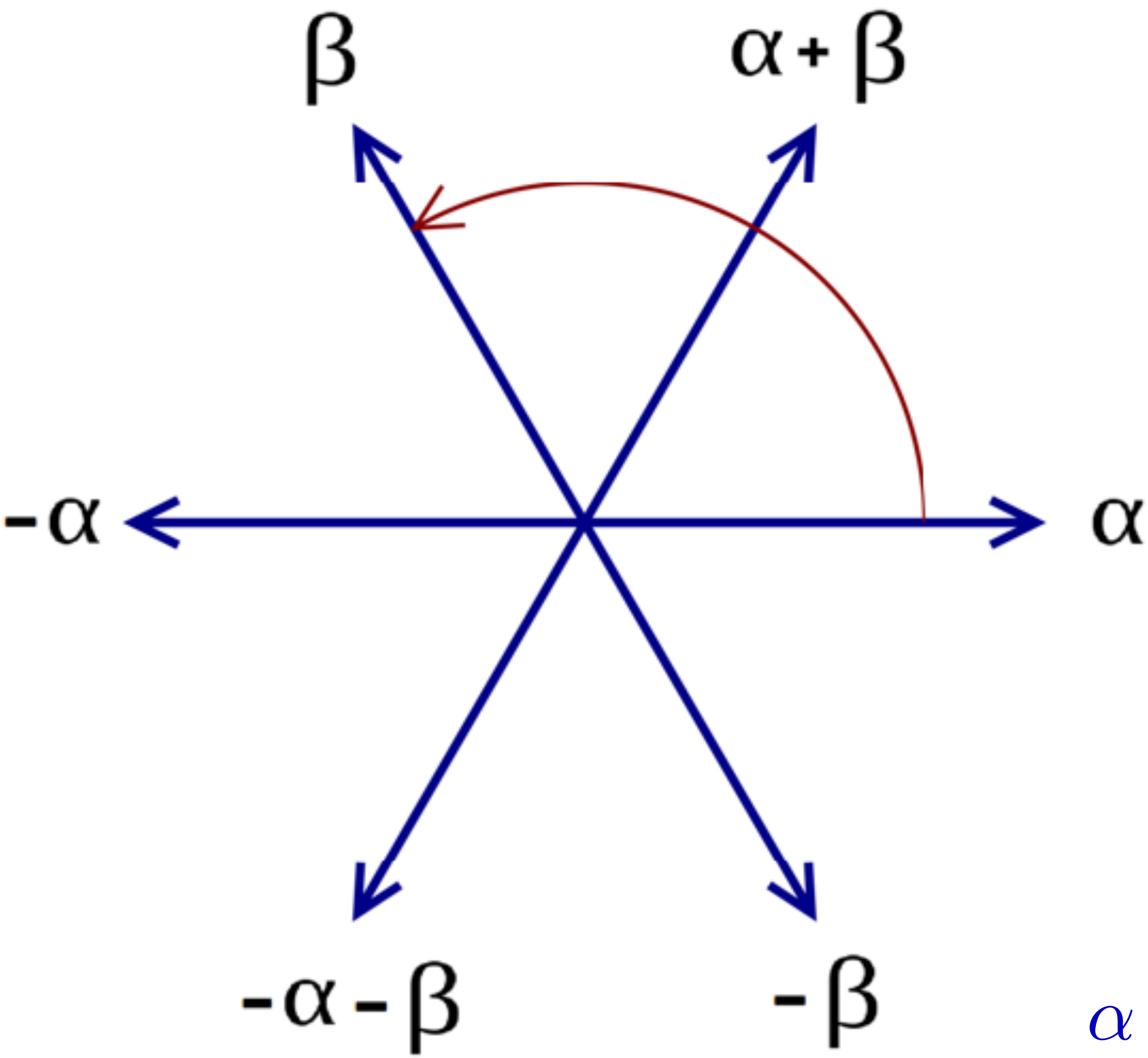


$\cdots$

monopole charges

α_1 α_2 α_3

Roots of SU(3)



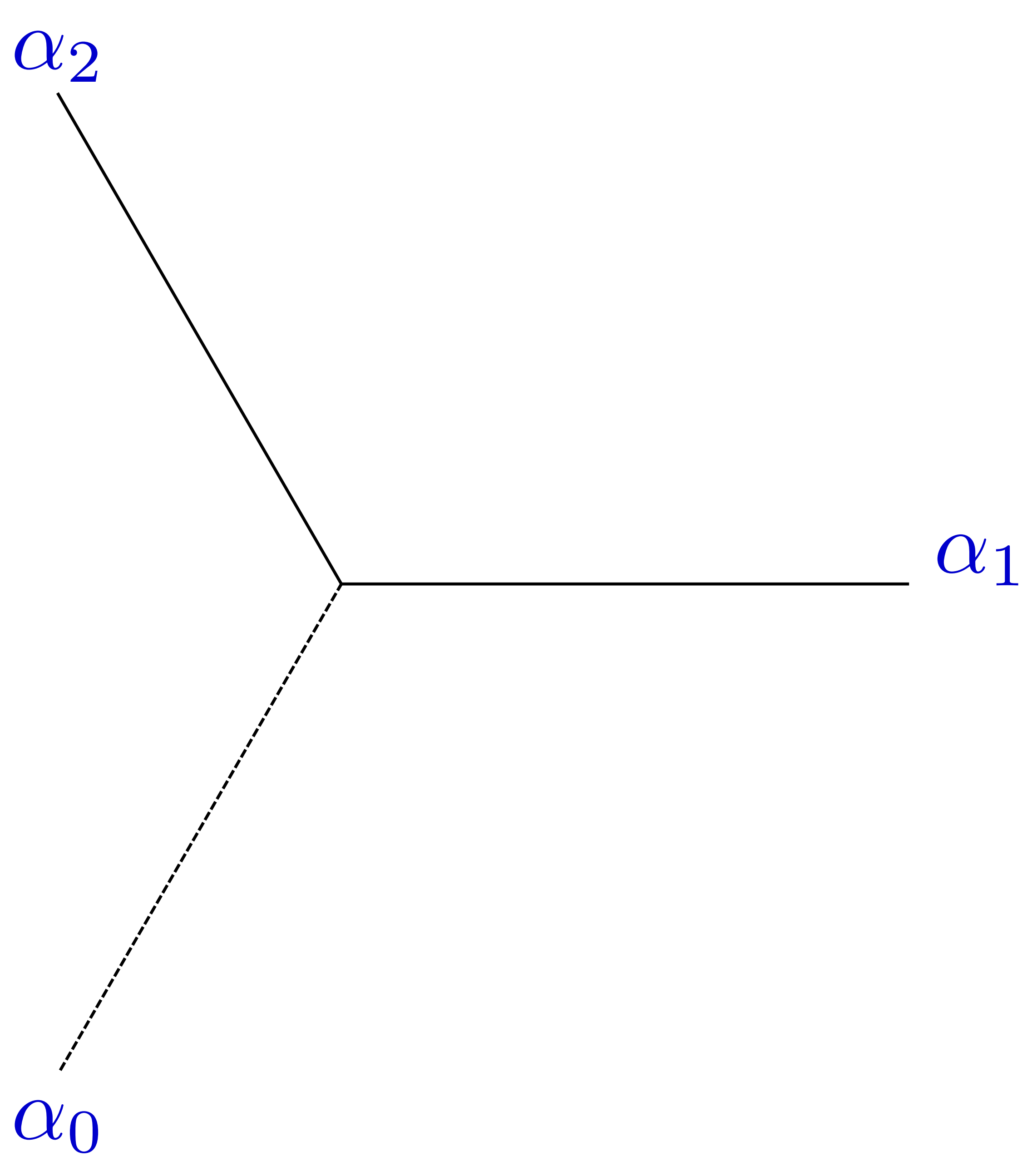
$$\mathbf{H} = (T^3, T^8)$$

$$\langle \phi \rangle = \mathbf{a} \cdot \mathbf{H}$$

$$\mathbf{a} = v_1 \alpha_1 + v_2 \beta$$

$$\alpha = (1, 0) \qquad \beta = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$$

Roots of SU(3)



$$\mathbf{H} = \left(T^3, T^8\right)$$

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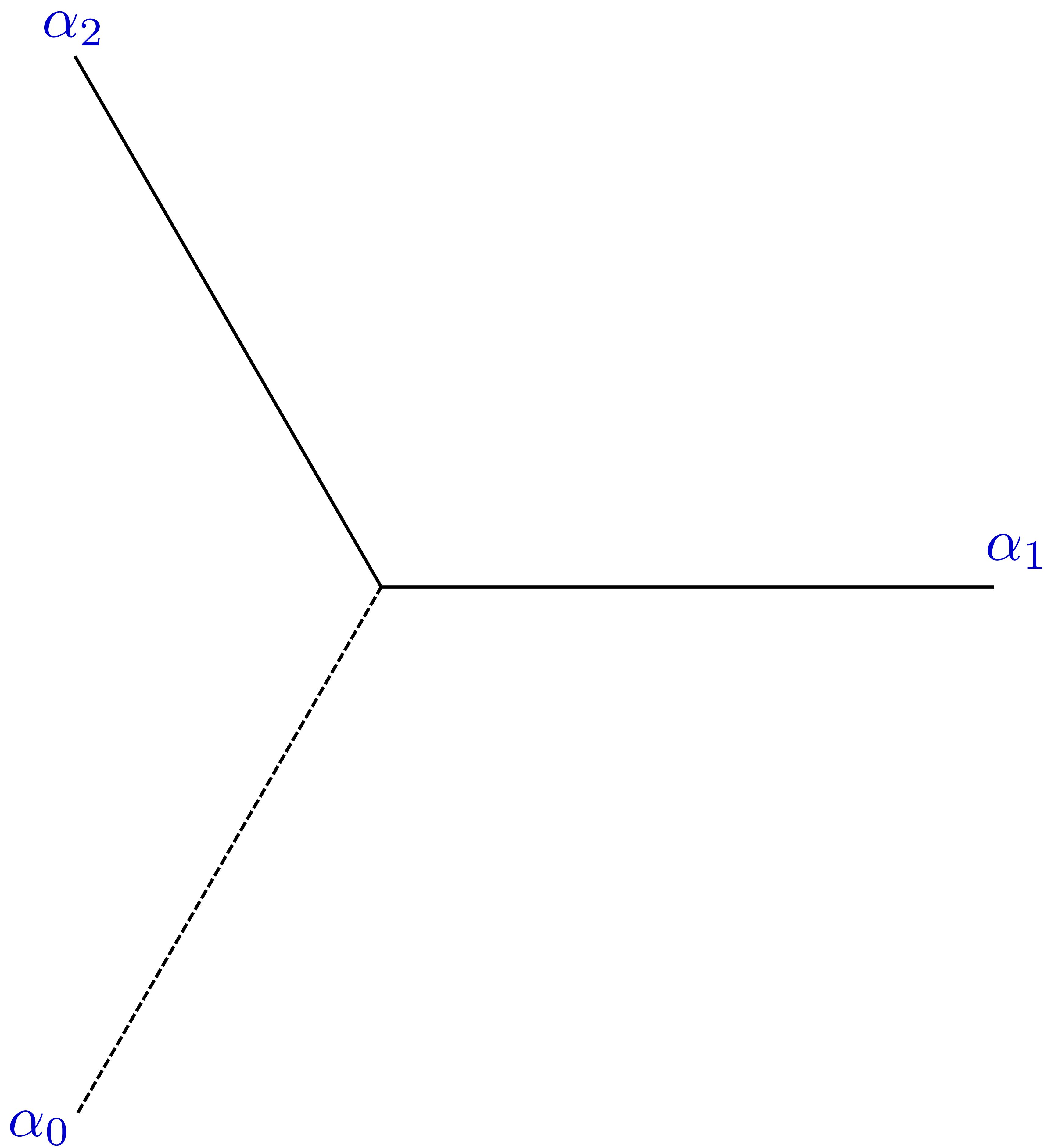

α_2
α_1
α_0

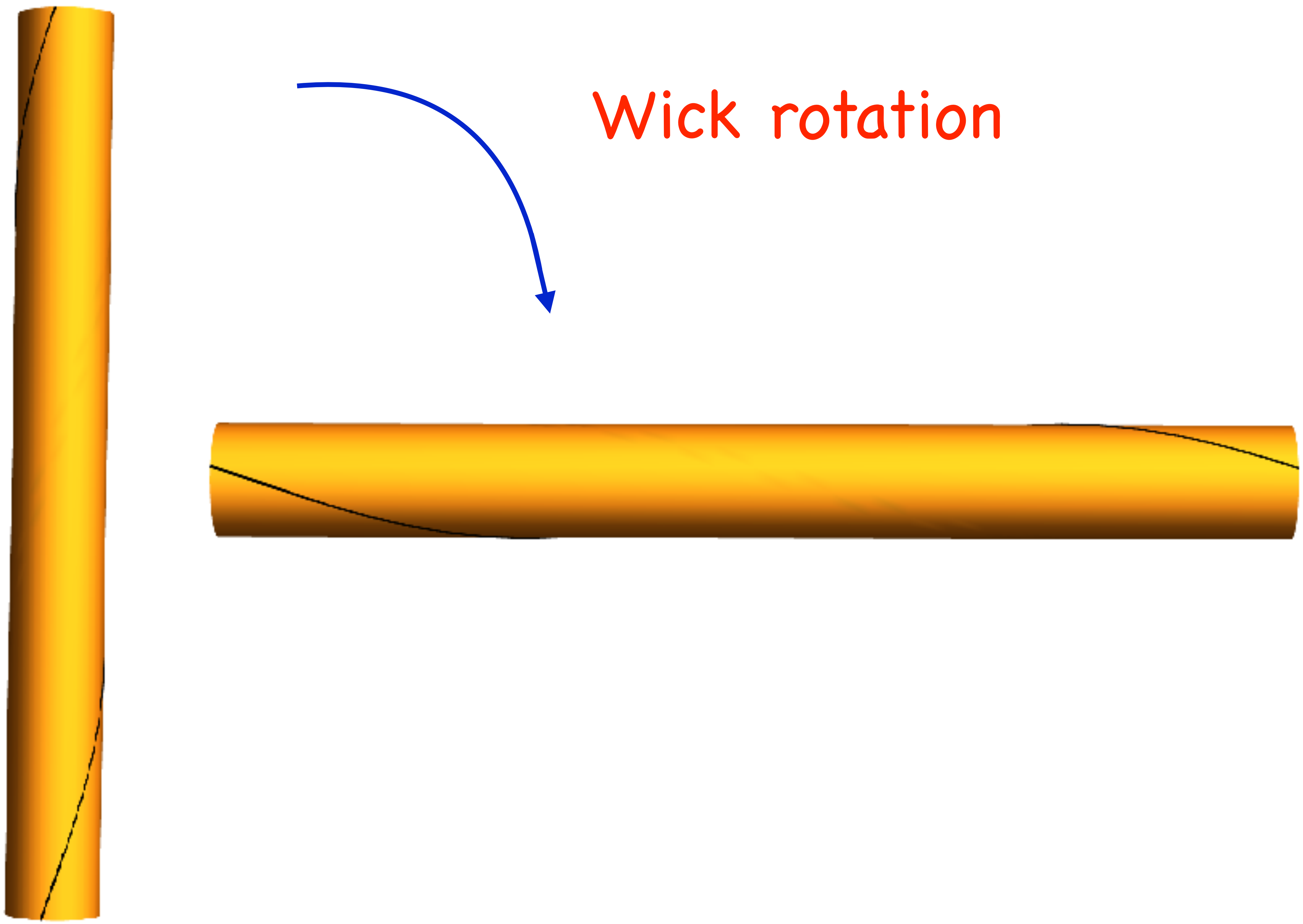
Monopole Solutions

$$\langle \phi \rangle = \mathbf{a} \cdot \mathbf{H} \qquad\qquad \mathbf{a} = v_1\, \alpha + v_2\, \beta$$

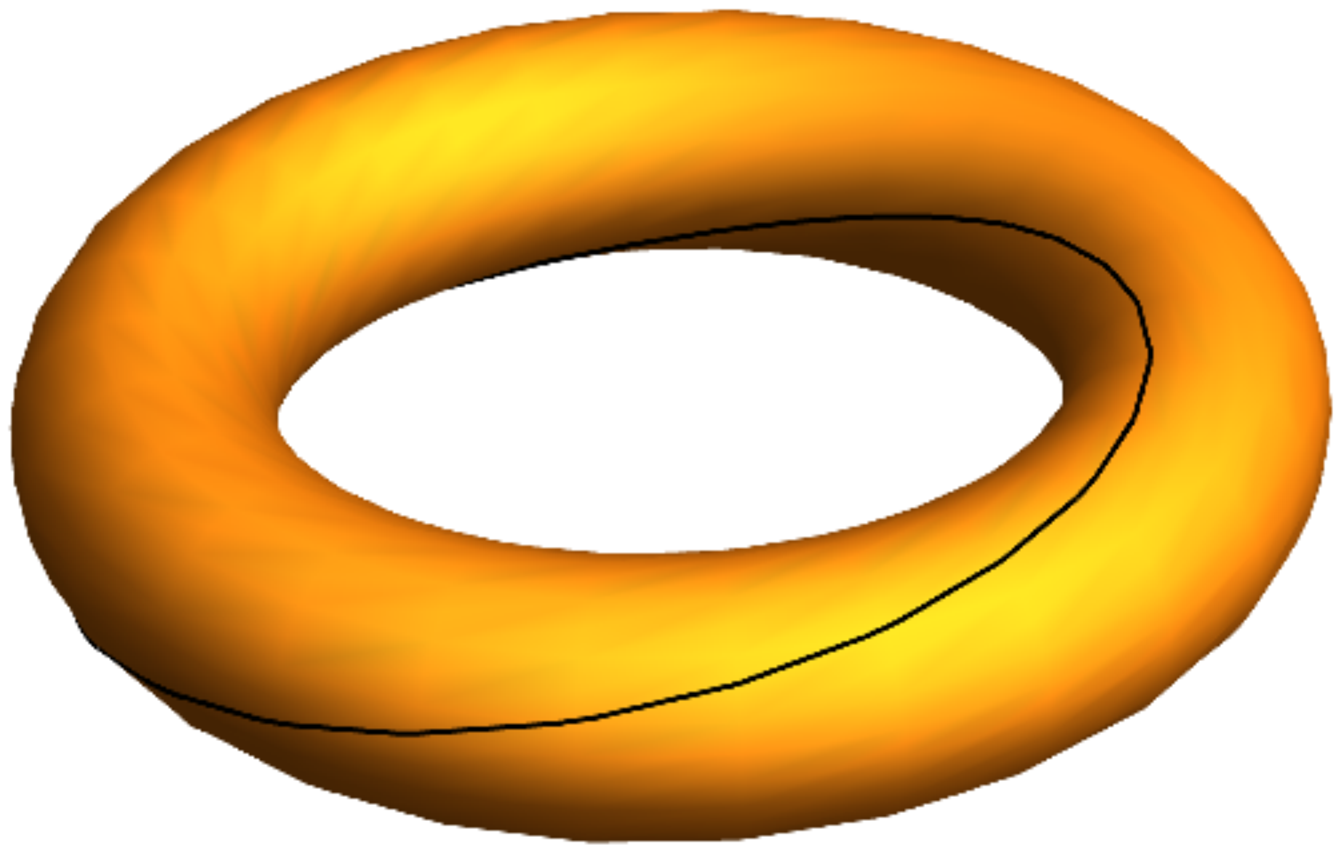
$$\phi = v_1\, \alpha \cdot H + \hat{r}^a T^a_\beta\, v_2\, h(v_2 r)\ ; \qquad T^3_\beta = \beta \cdot \mathbf{H}$$

$$\phi = v_2\, \beta \cdot H + \hat{r}^a T^a_\alpha\, v_1\, h(v_1 r)\ ; \qquad T^3_\alpha = \alpha \cdot \mathbf{H}$$

4D -> 3D x S^1



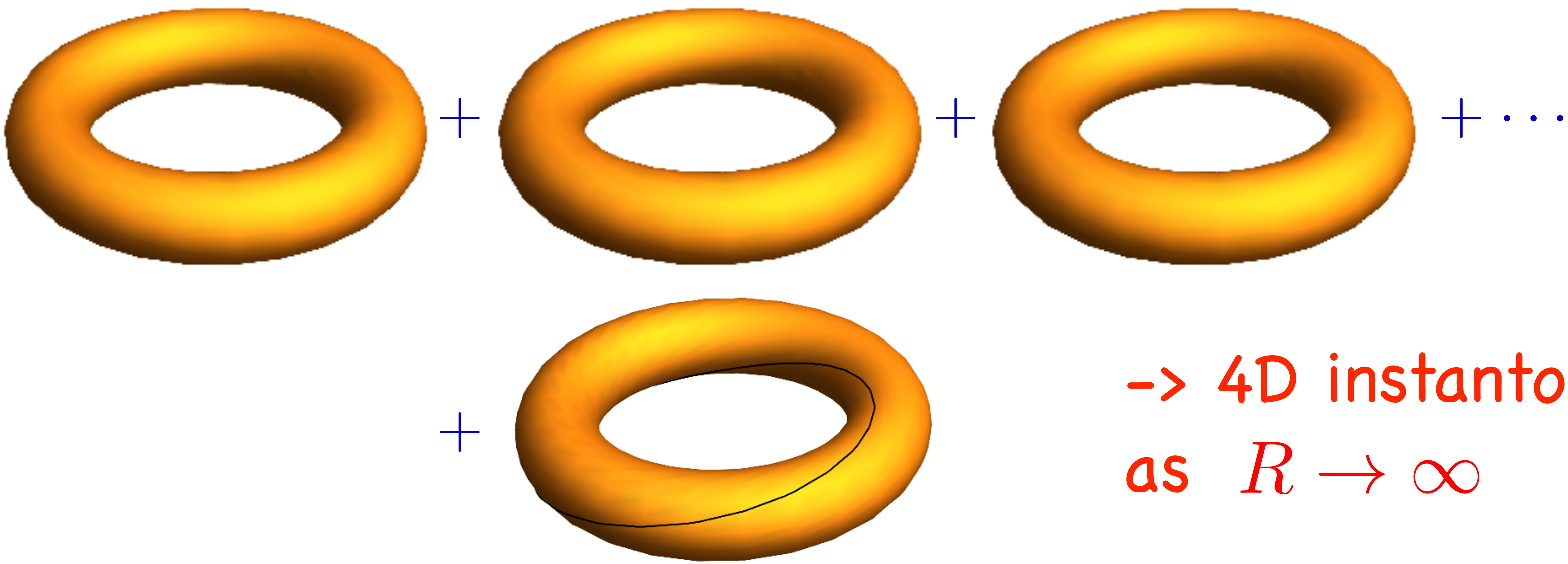
$4D \rightarrow 3D \times S^1$



KK monopole solution

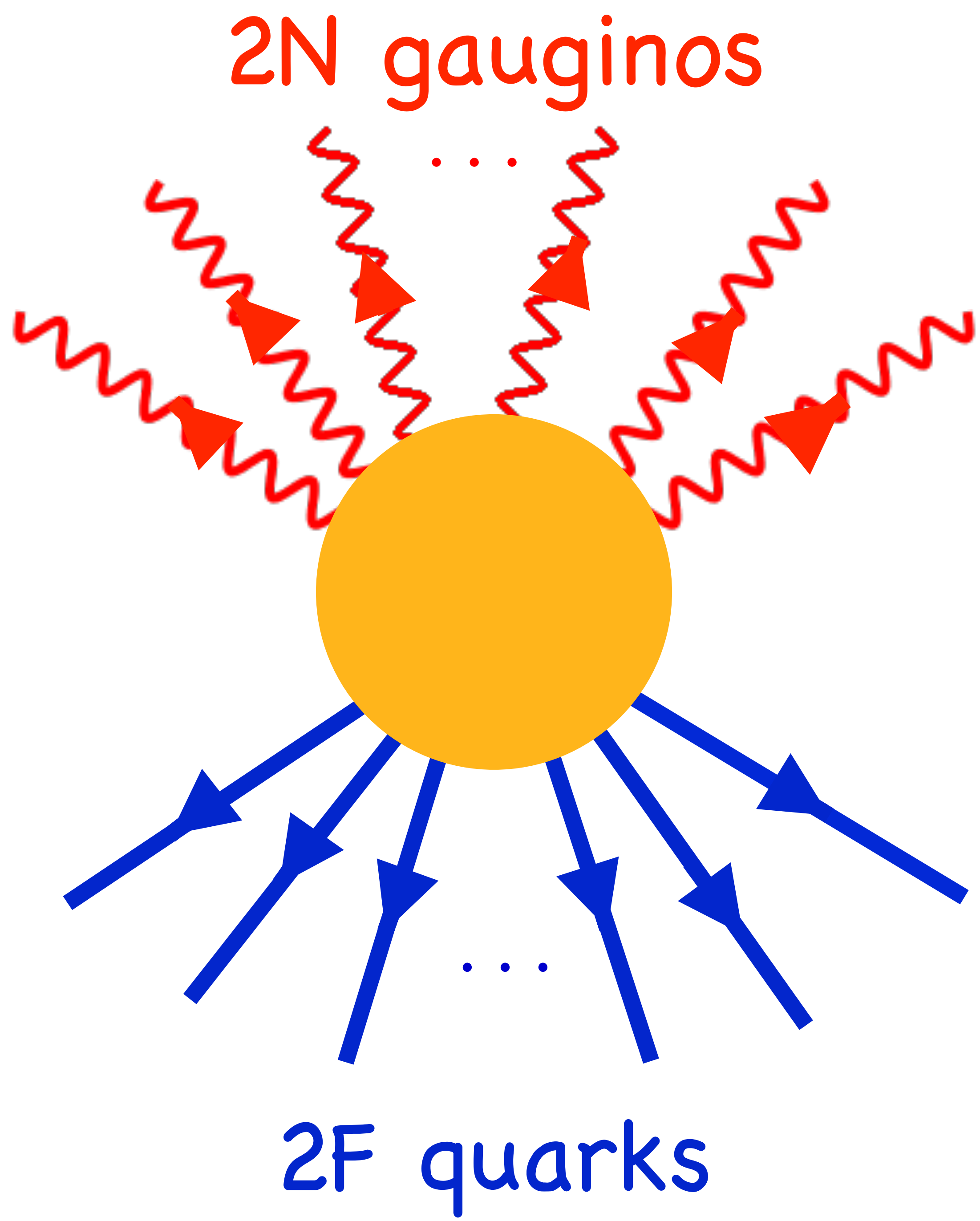
3D x S^1 -> 4D

N-1 monopole solutions + KK monopole

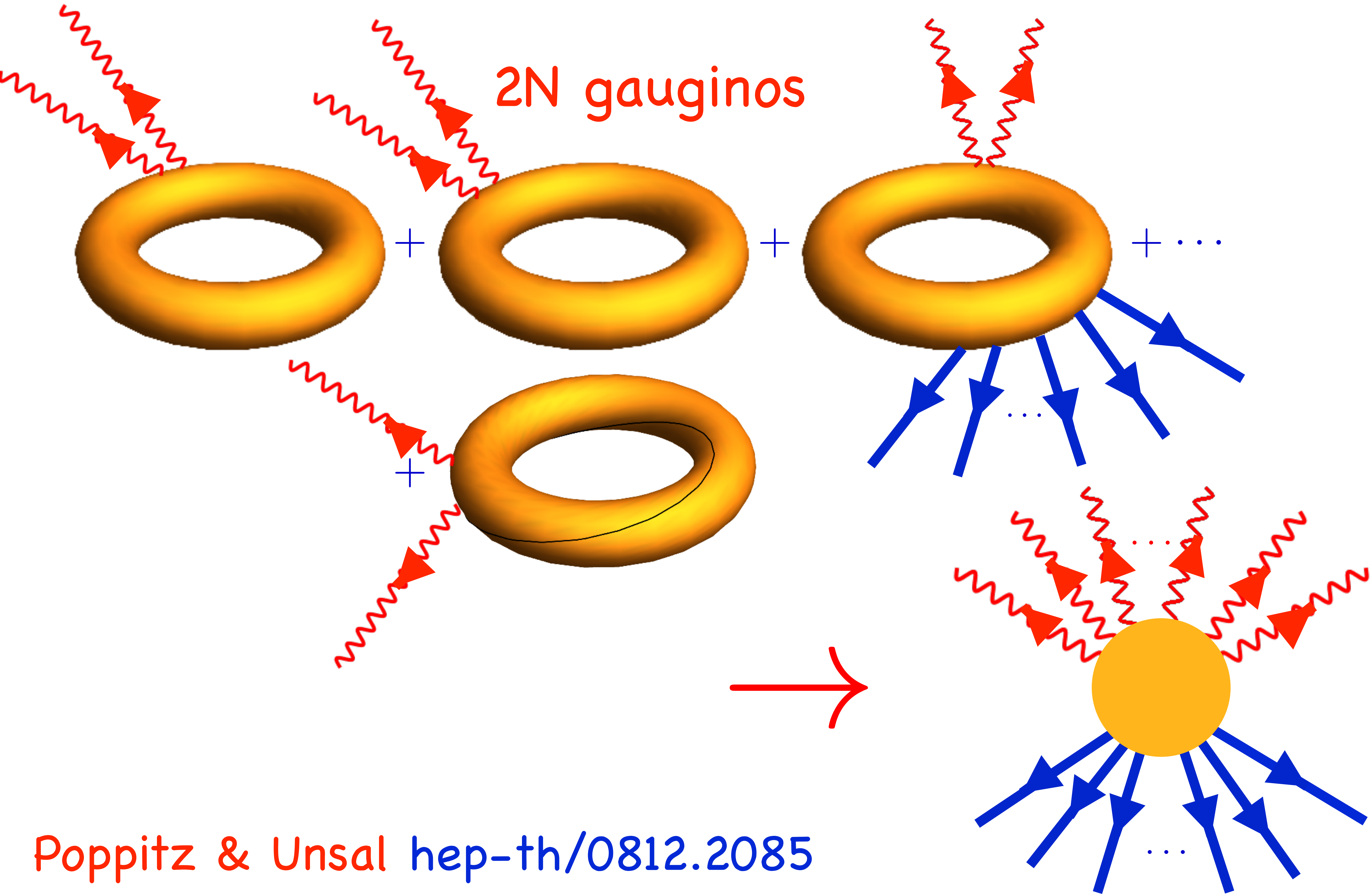


-> 4D instanton
as $R \to \infty$

Instanton Zero Modes

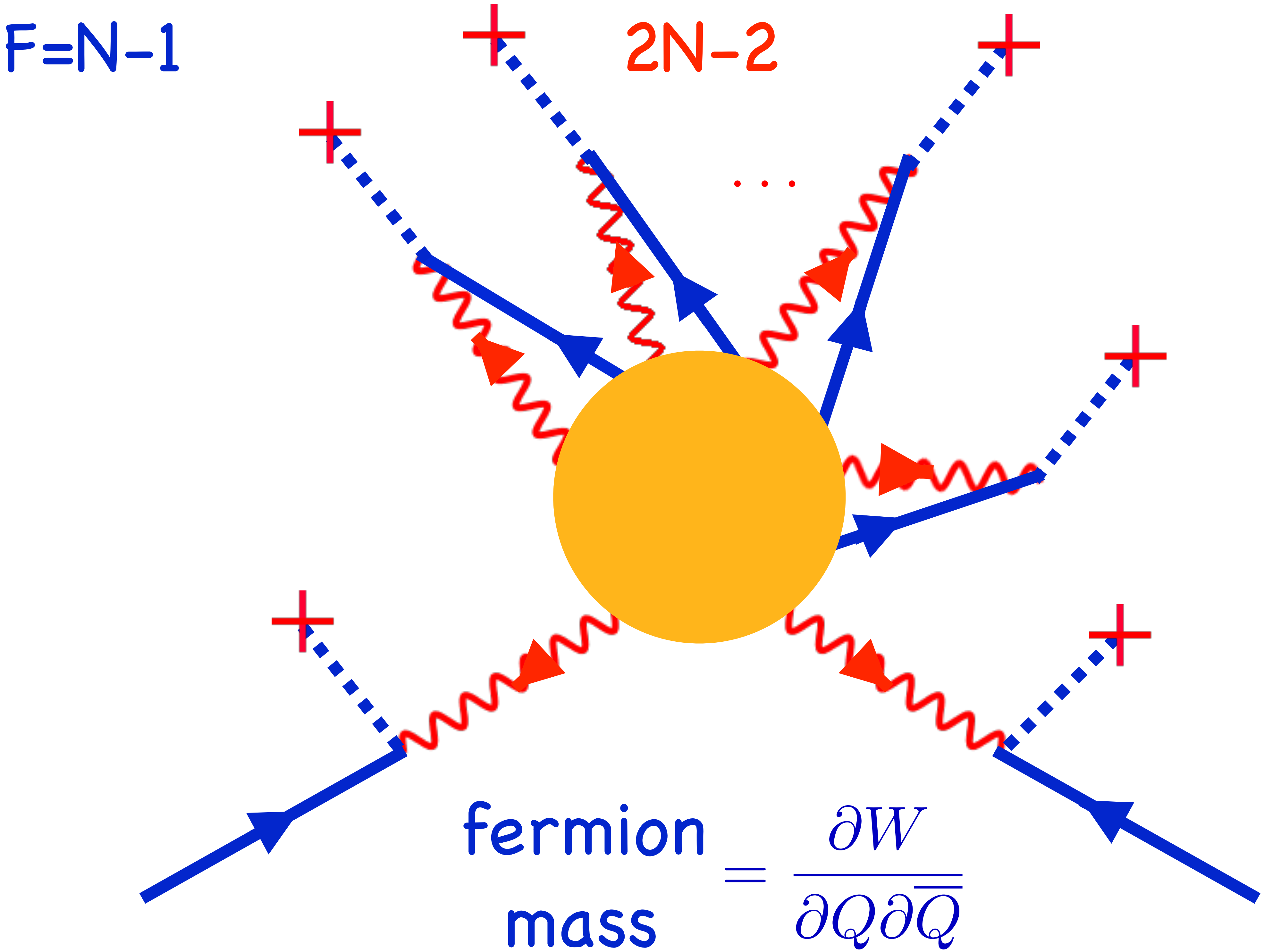


Instanton Zero Modes

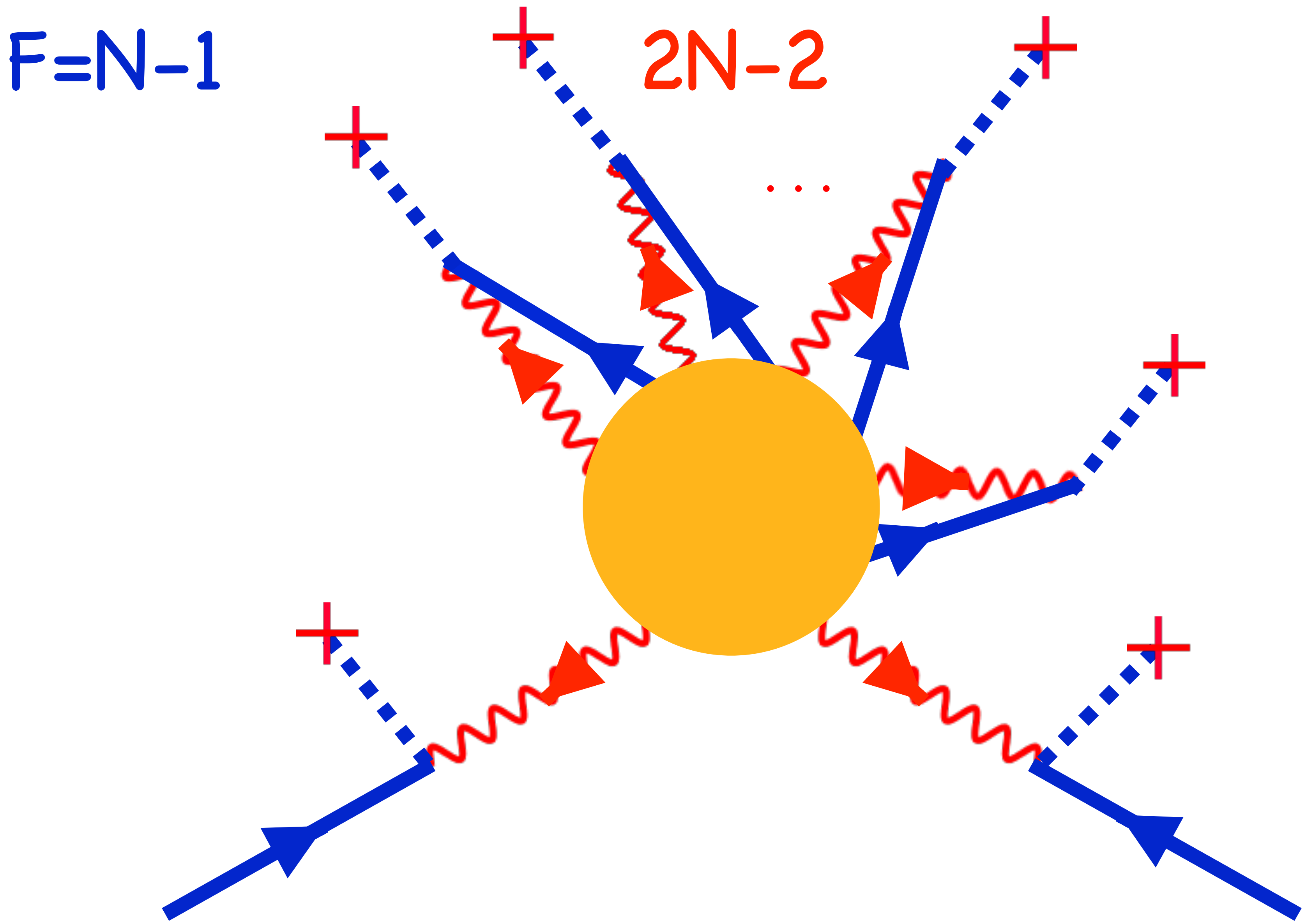


Poppitz & Unsal hep-th/0812.2085

Instanton Zero Modes



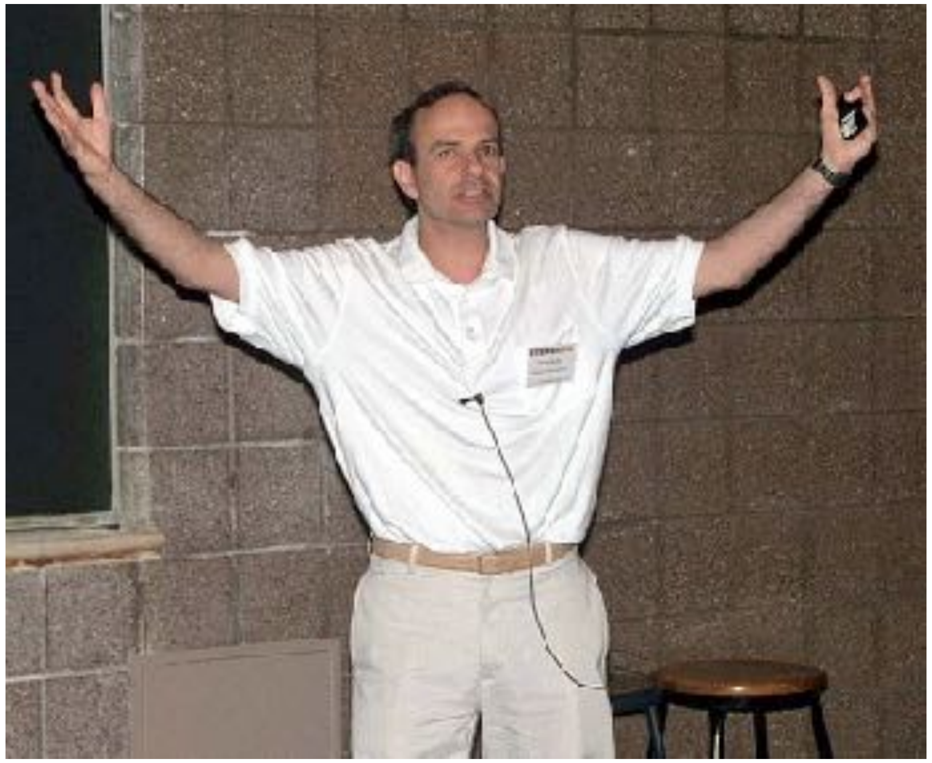
Instanton Superpotential



$$W = \frac{\Lambda^{3N-F} \det\, Q^* \overline{Q}^*}{|\det\, Q\overline{Q}|^2} = \frac{\Lambda^{3N-F}}{\det\, Q\overline{Q}}$$

Affleck-Dine Seiberg Superpotential

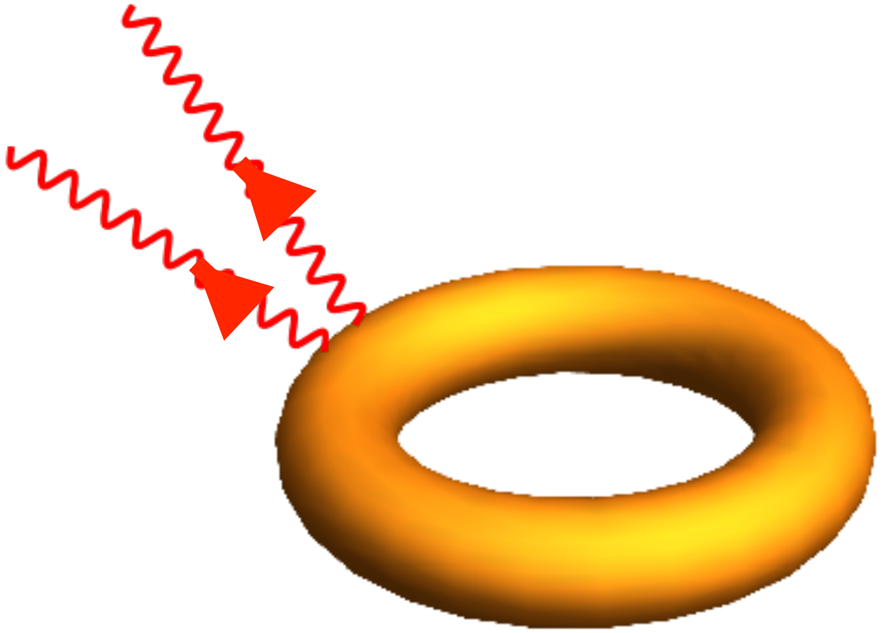




F < N $$W_{\text{ADS}} = (N-F)\left(\frac{\Lambda^{3N-F}}{\det Q\overline{Q}}\right)^{\frac{1}{N-F}}$$

where does this come from?

Affleck-Harvey-Witten



$R \to 0$

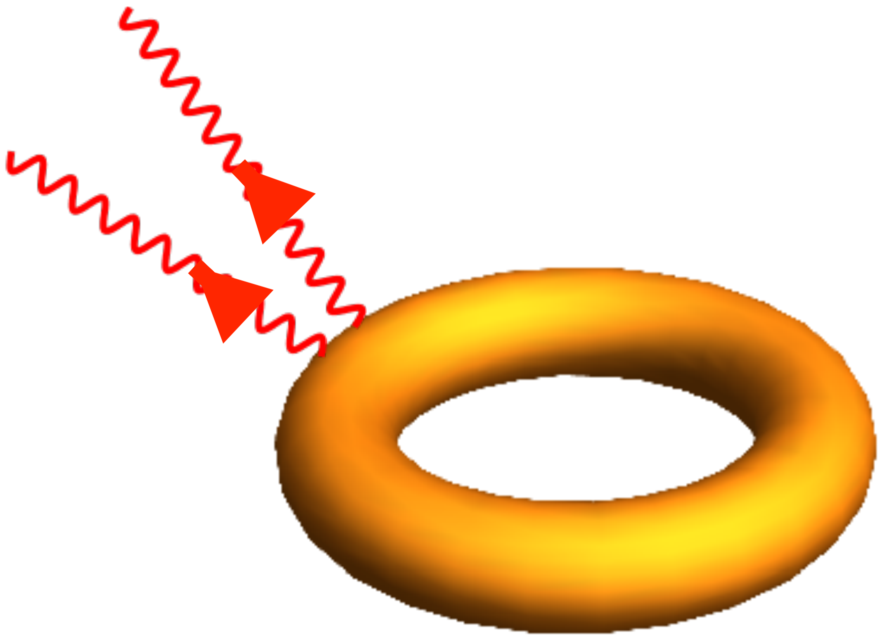
$$W_{3\mathrm{D}} = \sum_i \frac{1}{Y_i}$$

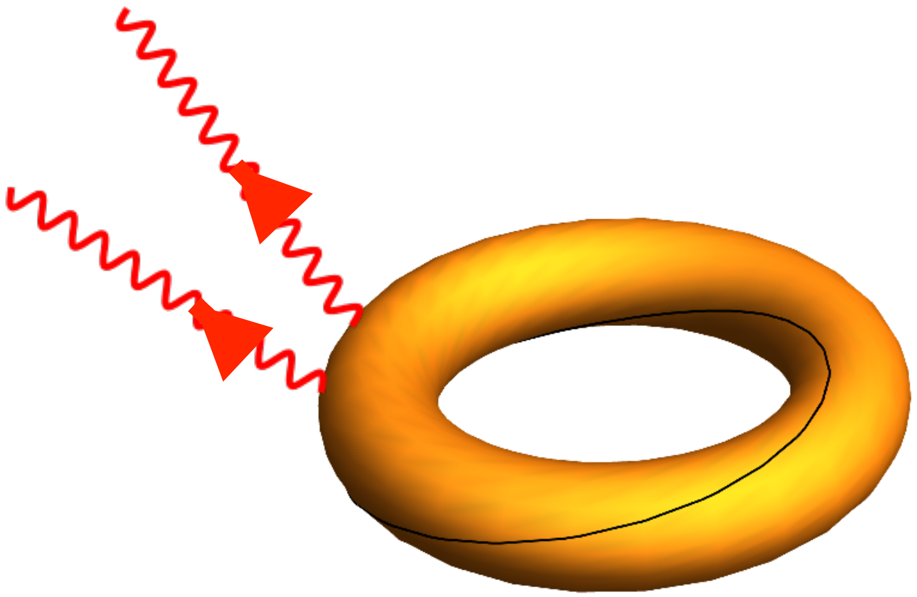
$$Y_i = e^{\mathbf{a}\cdot\alpha_i + i\gamma_i} \qquad \phi = \mathbf{a}\cdot\mathbf{H}$$

$$\partial_m \gamma_i = \epsilon_{mnp} F_i^{np}$$

Nucl. Phys. B206 (1982) 413

Finite R





$$W = \sum_i \frac{1}{Y_i} + \eta Y_{KK}$$

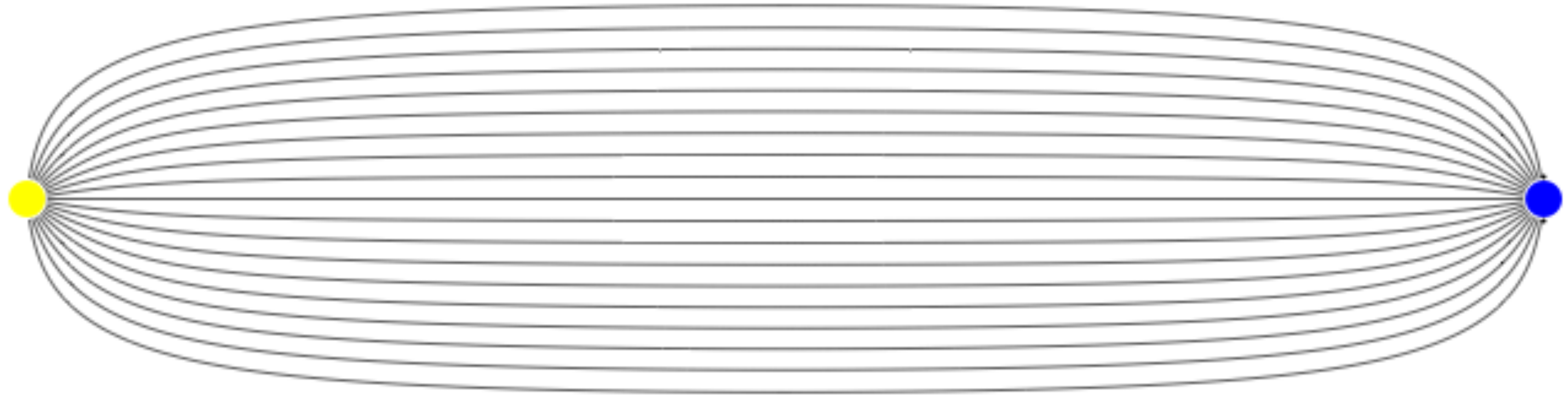
Mixed Coulomb Branch
SU(3) with F=1

$$\phi = \frac{1}{2}\mathrm{diag}(v, 0, -v) \qquad Q = \overline{Q} = \begin{pmatrix} 0 \\ q \\ 0 \end{pmatrix}$$

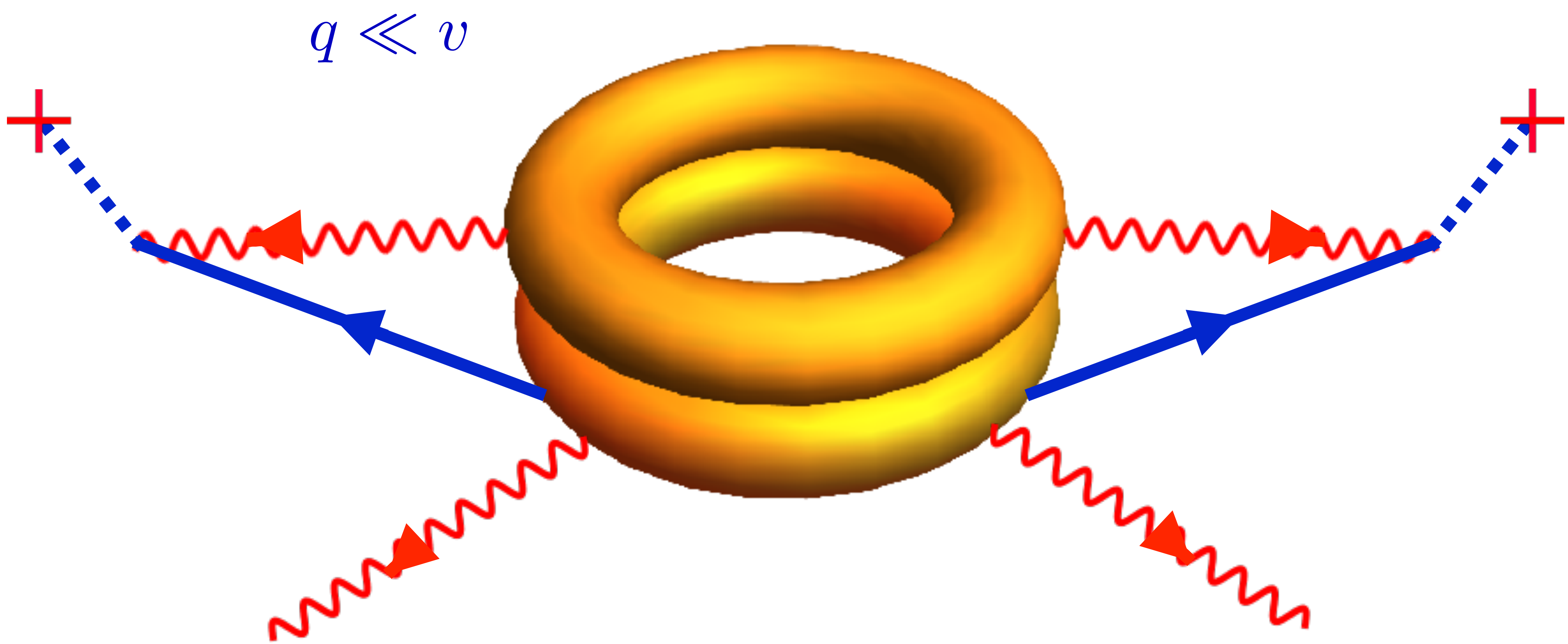
SU(3)->U(1)xU(1) SU(3)->SU(2)

SU(3)->U(1)

monopoles are confined



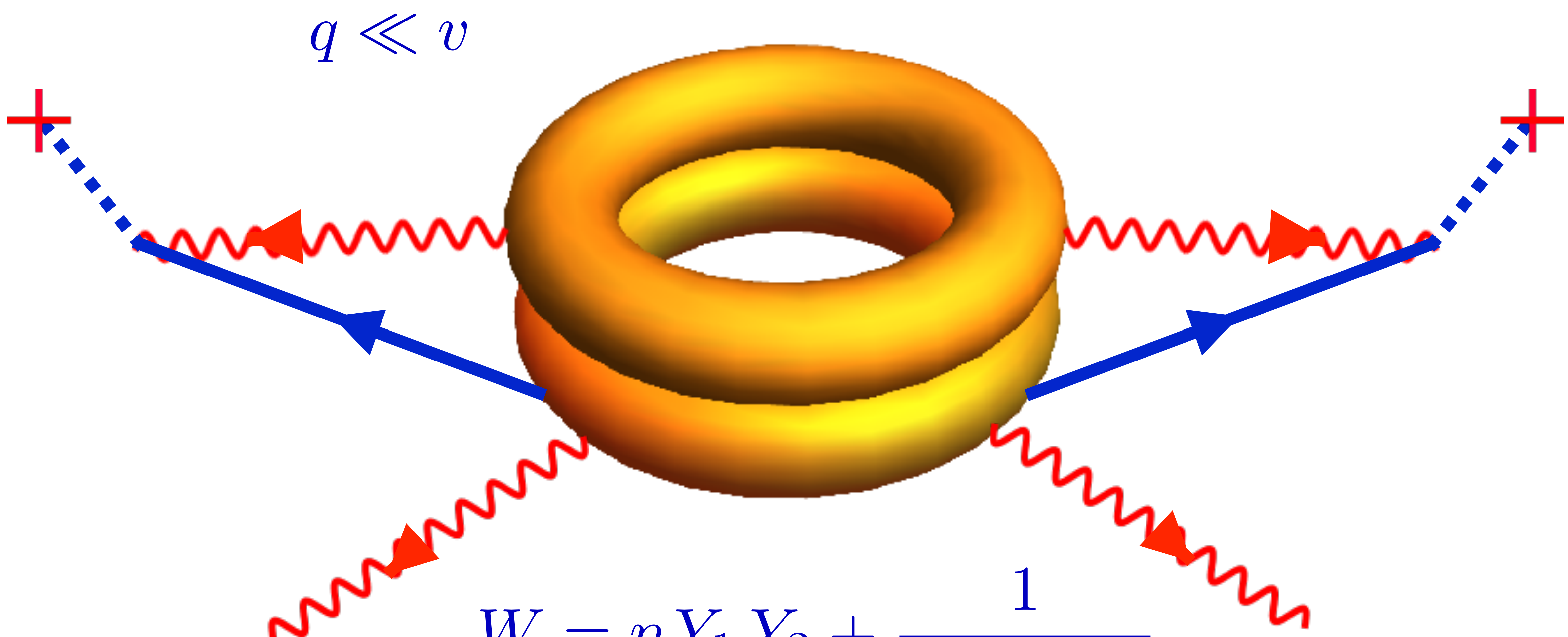
Mixed Coulomb Branch
SU(3) with F=1



monopoles are confined

superHiggs mechanism gives fermions masses

Mixed Coulomb Branch
SU(3) with F=1



$$W = \eta\, Y_1\, Y_2 + \frac{1}{Y_1\, Y_2\, Q\overline{Q}}$$

$$W = 2\left(\frac{\eta}{\det\, Q\overline{Q}}\right)^{\frac{1}{2}}$$

Mixed Coulomb Branch SU(3) with F=1

$$q \gg \frac{1}{R}, v$$

SU(3)->SU(2) in "4D", F=0

$$\phi = \mathbf{a} \cdot \mathbf{H}$$
$$\mathbf{a} = v(\alpha + \beta)$$

$$\Lambda^8 = \Lambda_L^6 q^2$$

$$W = \eta_L Y_L + \frac{1}{Y_L}$$

matches, since $Y_L \propto Y_1 Y_2 q^2$

$$\eta_L = \frac{\eta}{q^2}$$

SU(N) with F < N-1

ϕ has F zeros

$\downarrow$

SU(N)->SU(F)xU(1)$^{N-F}$

$Q, \overline{Q}$ have F VEVs

$\downarrow$

SU(N)->SU(N-F)

$\downarrow$

SU(N)->U(1)$^{N-F-1}$

F+1 monopoles are confined

2F gauginos get masses

2(F+1)-2F= 2

2 gaugino legs => ADS super potential

Conclusions

Monopoles are still fascinating
after all these years

Confined monopoles relate
3D BPS monopoles to
the 4D ADS superpotential