Quintessence, Unified Dark Energy and Dark Matter, and Gravity-Assisted Higgs Mechanism

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Based on and extending:

- E. Guendelman, E.Nissimov and S. Pacheva, *arxiv:1709.03786*.
- E. Guendelman, E.Nissimov and S. Pacheva, Bulgarian Journal of Physics 44 (2017) 15 (*arXiv:1609.06915*).
- E. Guendelman, E.Nissimov and S. Pacheva, *Int.J.Mod.Phys.* **D25** (2016) 1644008 (*arxiv:1603.06231*), *honorable mention in 2016 Gravity Research Foundation Competition for Essays on Gravitation*.
- E. Guendelman, E.Nissimov and S. Pacheva, *Euro. Phys. J.* C75 (2015) 472-479 (*arxiv:1508.02008*).
- E. Guendelman, E.Nissimov and S. Pacheva, *Euro. Phys. J.* C76 (2016) 90 (*arxiv:1511.07071*)
- E. Guendelman, R. Herrera, P. Labrana, E.Nissimov and S. Pacheva, *General Relativity and Gravitation* 47 (2015) art.10 (*arxiv:1408.5344v4*).

Introduction - Overview of Talk

Dark energy and **dark matter**, occupying around 70% and 25% of the matter content of the “late” (today’s) Universe, respectively, are still among most unexplained “mysteries” in cosmology and astrophysics.

- Dark energy is responsible for the accelerated expansion of today’s Universe, *i.e.*, dark energy acts effectively as **repulsion force** among the galaxies – completely counterintuitivly w.r.t. naive notion about gravity as an attractive force.
- And vice versa, dark matter holds together the matter objects inside the galaxies.

Both interact only gravitationally – no direct interaction with ordinary (baryonic) matter, in particular, they do not interact electromagnetically and thus remain “dark”.

Introduction - Overview of Talk

Principal challenge in modern cosmology is to understand theoretically from first principles the nature of both “dark” species as a manifestation of the dynamics of a single entity of matter.

Multitude of approaches proposed so far:

- “Chaplygin gas” models [Kamenshchik et.al., Bilic et.al];
- “purely kinetic k-essence” models [Scherrer,. . .];
- “Mimetic” dark matter model [Mukhanov et.al.].

We will achieve unified description of dark energy and dark matter based on a class of generalized non-canonical models of gravity interacting with a single scalar **“darkon”** field employing the **method of non-Riemannian volume-forms (volume elements)**.

The latter are constructed in terms of higher-rank gauge fields which are essentially pure-gauge degrees of freedom - *i.e.*, **NO additional gravitational degrees of freedom are introduced.**

Introduction - Overview of Talk

Next, we will also couple a second scalar **“inflaton”** field describing the universe’s evolution in a unified way (“quintessence”), as well as the fields of the electroweak bosonic sector.

Due to the remarkable impact of the non-Riemannian volume-forms we obtain:

- Scalar field effective potential possessing two infinitely large flat regions as function of the “inflaton” with vastly different heights (scales) corresponding to the “early” and “late” epochs of the Universe, respectively.
- **We obtain a gravity-assisted generation of electro-weak spontaneous gauge symmetry breaking in the post-inflationary “late” Universe, while the Higgs-like scalar remains massless in the “early” Universe.** This is explicit implementaion of an intriguing idea [Bekenstein-86].

First, Let us consider the following simple particular case of a non-conventional gravity-scalar-field action – a member of the general class of the “modified-measure” gravity-matter theories (for simplicity we use units with the Newton constant $G_N = 1/16\pi$):

$$S = \int d^4x \sqrt{-g}\, R + \int d^4x \big(\sqrt{-g} + \Phi(C)\big) L(u, Y) \; . \qquad (1)$$

Here R denotes the standard Riemannian scalar curvature for the pertinent Riemannian metric $g_{\mu\nu}$. In the second term in (1) – the scalar field Lagrangian is coupled *symmetrically* to two mutually independent spacetime volume-forms – the standard Riemannian $\sqrt{-g}$ and to an alternative non-Riemannian one:

$$\Phi(C) = \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu C_{\nu\kappa\lambda} \; . \qquad (2)$$

$L(u,Y)$ is general-coordinate invariant Lagrangian of a single scalar field $u(x)$, the simplest example being:

$$L(u,Y) = Y - V(u) \quad , \quad Y \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu u \partial_\nu u \; , \qquad (3)$$

Crucial new property – we obtain *dynamical constraint* on $L(u,Y)$ as a result of the equations of motion w.r.t. $C_{\mu\nu\lambda}$:

$$\partial_\mu L(u,Y) = 0 \;\longrightarrow\; L(u,Y) = -2M_0 = \mathrm{const} \; , \qquad (4)$$

i.e., $Y = V(u) - 2M_0$. M_0 will play the role of dynamically generated cosmological constant.

A second crucial property - *hidden strongly nonlinear Noether symmetry* of scalar field action in (1) due to the presence of $\Phi(C)$ (here below $\mathcal{C}^\mu \equiv \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda} C_{\nu\kappa\lambda}$):

$$\delta_\epsilon u = \epsilon \sqrt{Y} \quad , \quad \delta_\epsilon g_{\mu\nu} = 0 \quad , \quad \delta_\epsilon \mathcal{C}^\mu = -\epsilon \frac{1}{2\sqrt{Y}} g^{\mu\nu} \partial_\nu u \big(\Phi(C) + \sqrt{-g}\big) \; . \qquad (5)$$

Hidden Noether Symmetry and Unification of DE and DM

Then, standard Noether procedure yields a conserved current:

$$\nabla_\mu J^\mu = 0 \quad , \quad J^\mu \equiv -\left(1 + \frac{\Phi(C)}{\sqrt{-g}}\right)\sqrt{2Y}\, g^{\mu\nu}\partial_\nu u \tag{6}$$

The energy-momentum tensor $T_{\mu\nu}$ and J^μ (6) can be cast into a relativistic hydrodynamical form (taking into account (4)):

$$T_{\mu\nu} = -2M_0 g_{\mu\nu} + \rho_0 u_\mu u_\nu \quad , \quad J^\mu = \rho_0 u^\mu \; , \tag{7}$$

where the pressure $p = -2M_0 = \mathrm{const}$ and:

$$\rho_0 \equiv \left(1 + \frac{\Phi(C)}{\sqrt{-g}}\right) 2Y \quad , \quad u_\mu \equiv -\frac{\partial_\mu u}{\sqrt{2Y}} \quad , \quad u^\mu u_\mu = -1 \; . \tag{8}$$

The total energy density is $\rho = \rho_0 - p = 2M_0 + \left(1 + \frac{\Phi(C)}{\sqrt{-g}}\right) 2Y$.

Hidden Noether Symmetry and Unification of DE and DM

Because of the constant pressure ($p = -2M_0$) $\nabla^\nu T_{\mu\nu} = 0$ implies *both* hidden Noether symmetry current $J^\mu = \rho_0 u^\mu$ conservation, as well as *geodesic fluid motion*:

$$\nabla_\mu \left(\rho_0 u^\mu\right) = 0 \quad , \quad u_\nu \nabla^\nu u_\mu = 0 \; . \tag{9}$$

Therefore, $T_{\mu\nu} = -2M_0 g_{\mu\nu} + \rho_0 u_\mu u_\nu$ represents an exact sum of two contributions of the two dark species:

$$p = p_{\mathrm{DE}} + p_{\mathrm{DM}} \quad , \quad \rho = \rho_{\mathrm{DE}} + \rho_{\mathrm{DM}} \tag{10}$$

$$p_{\mathrm{DE}} = -2M_0 \;\; , \;\; \rho_{\mathrm{DE}} = 2M_0 \quad ; \quad p_{\mathrm{DM}} = 0 \;\; , \;\; \rho_{\mathrm{DM}} = \rho_0 \; , \tag{11}$$

i.e., the dark matter component is a dust fluid flowing along geodesics. **This is explicit unification of dark energy and dark matter originating from the dynamics of a single scalar field - the “darkon” u.**

Further developments along this line in [E. Guendelman’s talk].

“Quintessence”, Unified DE/DM and Higgs

We will now extend our previous gravity-“darkon” model to gravity coupled to both “inflaton” $\varphi(x)$ and “darkon” $u(x)$ scalar fields within the non-Riemannian volume-form formalism, as well as we will also add coupling to the bosonic sector of the electro-weak model:

$$S = \int d^4x\, \Phi(A)\Big[g^{\mu\nu} R_{\mu\nu}(\Gamma) + L_1(\varphi, X) + L_2(\sigma, \nabla\sigma; \varphi)\Big] + \int d^4x\, \Phi(B)\Big[U(\varphi) + L_3(\mathcal{A}, \mathcal{B}) + \frac{\Phi(H)}{\sqrt{-g}}\Big] + \int d^4x \big(\sqrt{-g} + \Phi(C)\big) L(u, Y)\,. \quad (12)$$

Here the following notations are used:

- $\Phi(A) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda}$ and $\Phi(B) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda}$ – two new independent non-Riemannian volume-forms (non-Riemannian volume elements) apart from $\Phi(C)$;

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- $\Phi(H) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu H_{\nu\kappa\lambda}$ is the dual field-strength of an additional auxiliary tensor gauge field $H_{\nu\kappa\lambda}$ crucial for the consistency of (12).
- Important - we use Palatini formalism: $R = g^{\mu\nu}R_{\mu\nu}(\Gamma)$; $g_{\mu\nu}$, $\Gamma^\lambda_{\mu\nu}$ – metric and affine connection are *apriori* independent.
- $\sigma \equiv (\sigma_a)$ is a complex $SU(2)\times U(1)$ iso-doublet Higgs-like scalar field with a Lagrangian:

$$L_2(\sigma, \nabla\sigma; \varphi) = -g^{\mu\nu}\left(\nabla_\mu\sigma_a\right)^*\nabla_\nu\sigma_a - V_0(\sigma)e^{\alpha\varphi} \; . \qquad (13)$$

The gauge-covariant derivative acting on σ reads:

$$\nabla_\mu\sigma = \Big(\partial_\mu - \frac{i}{2}\tau_A\mathcal{A}_\mu^A - \frac{i}{2}\mathcal{B}_\mu\Big)\sigma \; , \qquad (14)$$

with $\frac{1}{2}\tau_A$ (τ_A – Pauli matrices, $A=1,2,3$) indicating the $SU(2)$ generators.

- The “bare” σ-field potential is of the same form as the standard Higgs potential:

$$V_0(\sigma) = \frac{\lambda}{4}\left((\sigma_a)^*\sigma_a - \mu^2\right)^2 \ . \tag{15}$$

- The $SU(2)\times U(1)$ gauge field action $L(\mathcal{A},\mathcal{B})$ is of the standard Yang-Mills form (all $SU(2)$ indices $A,B,C=(1,2,3)$):

$$L_3(\mathcal{A},\mathcal{B}) = -\frac{1}{4g^2}F^2(\mathcal{A}) - \frac{1}{4g'^2}F^2(\mathcal{B}) \ , \tag{16}$$

$$F^2(\mathcal{A}) \equiv F^A_{\mu\nu}(\mathcal{A})F^A_{\kappa\lambda}(\mathcal{A})g^{\mu\kappa}g^{\nu\lambda} \ , \quad F^2(\mathcal{B}) \equiv F_{\mu\nu}(\mathcal{B})F_{\kappa\lambda}(\mathcal{B})g^{\mu\kappa}g^{\nu\lambda} \ ,$$

$$F^A_{\mu\nu}(\mathcal{A}) = \partial_\mu\mathcal{A}^A_\nu - \partial_\nu\mathcal{A}^A_\mu + \epsilon^{ABC}\mathcal{A}^B_\mu\mathcal{A}^C_\nu \ , \quad F_{\mu\nu}(\mathcal{B}) = \partial_\mu\mathcal{B}_\nu - \partial_\nu\mathcal{B}_\mu \ .$$

$\mathcal{A}^A_\mu$ ($A=1,2,3$) and $\mathcal{B}_\mu$ denote the corresponding $SU(2)$ and $U(1)$ electroweak gauge fields.

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- The “inflaton” φ Lagrangian terms are given by:

$$L_1(\varphi, X) = X - V_1(\varphi) \quad , \quad X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \; , \tag{17}$$

$$V_1(\varphi) = f_1 \exp\{\alpha\varphi\} \quad , \quad U(\varphi) \equiv f_2 \exp\{2\alpha\varphi\} \; , \tag{18}$$

where α, f_1, f_2 are dimensionful positive parameters.

- The form of the action (12) is fixed by the requirement of invariance under global Weyl-scale transformations:

$$g_{\mu\nu} \to \lambda g_{\mu\nu} \; , \; \Gamma^\mu_{\nu\lambda} \to \Gamma^\mu_{\nu\lambda} \; , \; \varphi \to \varphi - \frac{1}{\alpha} \ln \lambda \; ,$$
$$A_{\mu\nu\kappa} \to \lambda A_{\mu\nu\kappa} \; , \; B_{\mu\nu\kappa} \to \lambda^2 B_{\mu\nu\kappa} \; , \; H_{\mu\nu\kappa} \to H_{\mu\nu\kappa} \; , \tag{19}$$

and the electro-weak sector $(\sigma, \mathcal{A}, \mathcal{B})$ is inert w.r.t. (19).

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Eqs. of motion w.r.t. affine connection $\Gamma^{\mu}_{\nu\lambda}$ yield a solution for the latter as a Levi-Civita connection:

$$\Gamma^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\nu\lambda}(\bar{g}) = \frac{1}{2}\bar{g}^{\mu\kappa}\left(\partial_{\nu}\bar{g}_{\lambda\kappa} + \partial_{\lambda}\bar{g}_{\nu\kappa} - \partial_{\kappa}\bar{g}_{\nu\lambda}\right) \; , \qquad (20)$$

w.r.t. to the **Weyl-rescaled metric** $\bar{g}_{\mu\nu}$:

$$\bar{g}_{\mu\nu} = \chi_1 g_{\mu\nu} \quad , \quad \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}} \; . \qquad (21)$$

Transition from original metric $g_{\mu\nu}$ to $\bar{g}_{\mu\nu}$: **“Einstein-frame”**, where the gravity eqs. of motion are written in the standard form of Einstein’s equations: $R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) = \frac{1}{2}T^{\mathrm{eff}}_{\mu\nu}$ with an appropriate **effective** energy-momentum tensor given in terms of an Einstein-frame matter Lagrangian L_{eff} (see (25) below).

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Solutions of the eqs. of motion of the action (12) w.r.t. auxiliary tensor gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda}$ yield:

$$\frac{\Phi(B)}{\sqrt{-g}} \equiv \chi_2 = \mathrm{const} \quad , \quad R + L_1(\varphi, X) + L_2(\sigma, \nabla\sigma; \varphi) = M_1 = \mathrm{const} \; ,$$

$$U(\varphi) + L_3(\mathcal{A}, \mathcal{B}) + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \mathrm{const} \; . \quad (22)$$

Here M_1 and M_2 are arbitrary dimensionful and χ_2 arbitrary dimensionless integration constants, similar to M_0 (4).

Within the canonical Hamilton formalism we have shown that $M_0,\, M_{1,2},\, \chi_2$ are the only remnant of the auxiliary gauge fields $C_{\mu\nu\lambda},\, A_{\mu\nu\lambda},\, B_{\mu\nu\lambda},\, H_{\mu\nu\lambda}$ entering (12) – they have the meaning of conserved Dirac-constrained canonical momenta conjugated to some of the components of the latter.

We derive from (12) the physical *Einstein-frame* theory w.r.t. Weyl-rescaled Einstein-frame metric $\bar{g}_{\mu\nu}$ (21) and perform an additional "darkon" field redefinition $u \to \widetilde{u}$:

$$\frac{\partial \widetilde{u}}{\partial u} = \big(V_1(u) - 2M_0\big)^{-\frac{1}{2}} \quad ; \quad Y \to \widetilde{Y} = -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu \widetilde{u}\partial_\nu \widetilde{u} \; . \tag{23}$$

The Einstein-frame action reads:

$$S = \int d^4x \sqrt{-\bar{g}} \Big[R(\bar{g}) + L_{\rm eff}\big(\varphi, \bar{X}, \widetilde{Y}; \sigma, \bar{X}_\sigma, \mathcal{A}, \mathcal{B}\big) \Big] \; , \tag{24}$$

where (now the kinetic terms are given in terms of the Einstein-frame metric, *e.g.* $\bar{X} = -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$, etc.):

$$L_{\rm eff}\big(\varphi, \bar{X}, \widetilde{Y}; \sigma, \bar{X}_\sigma, \mathcal{A}, \mathcal{B}\big) = \bar{X} - \widetilde{Y}\Big(V_1(\varphi) + V_0(\sigma)e^{\alpha\varphi} + M_1\Big)$$
$$+ \widetilde{Y}^2 \Big[\chi_2 (U(\varphi) + M_2) - 2M_0\Big] + L[\sigma, \bar{X}_\sigma, \mathcal{A}, \mathcal{B}] \; , \tag{25}$$

with $L[\sigma, \bar{X}_\sigma, \mathcal{A}, \mathcal{B}] \equiv -\bar{g}^{\mu\nu}\big(\nabla_\mu \sigma_a\big)^* \nabla_\nu \sigma_a - \frac{\chi_2}{4g^2}\bar{F}^2(\mathcal{A}) - \frac{\chi_2}{4g'^2}\bar{F}^2(\mathcal{B})$.

“Quintessence”, Unified DE/DM and Higgs

For static (spacetime idependent) scalar field configurations we obtain from (25) the following Einstein-frame effective scalar “inflaton+Higgs” effective potential:

$$
\begin{aligned}
U_{\mathrm{eff}}(\varphi,\sigma) &= \frac{\Big(V_1(\varphi) + V_0(\sigma)e^{\alpha\varphi} + M_1\Big)^2}{4\big[\chi_2(U(\varphi)+M_2) - 2M_0\big]} \\
&= \frac{\Big[\Big(f_1 + \frac{\lambda}{4}\left((\sigma_a)^*\sigma_a - \mu^2\right)^2\Big)e^{\alpha\varphi} + M_1\Big]^2}{4\big[\chi_2(f_2 e^{2\alpha\varphi} + M_2) - 2M_0\big]}\,. \qquad (26)
\end{aligned}
$$

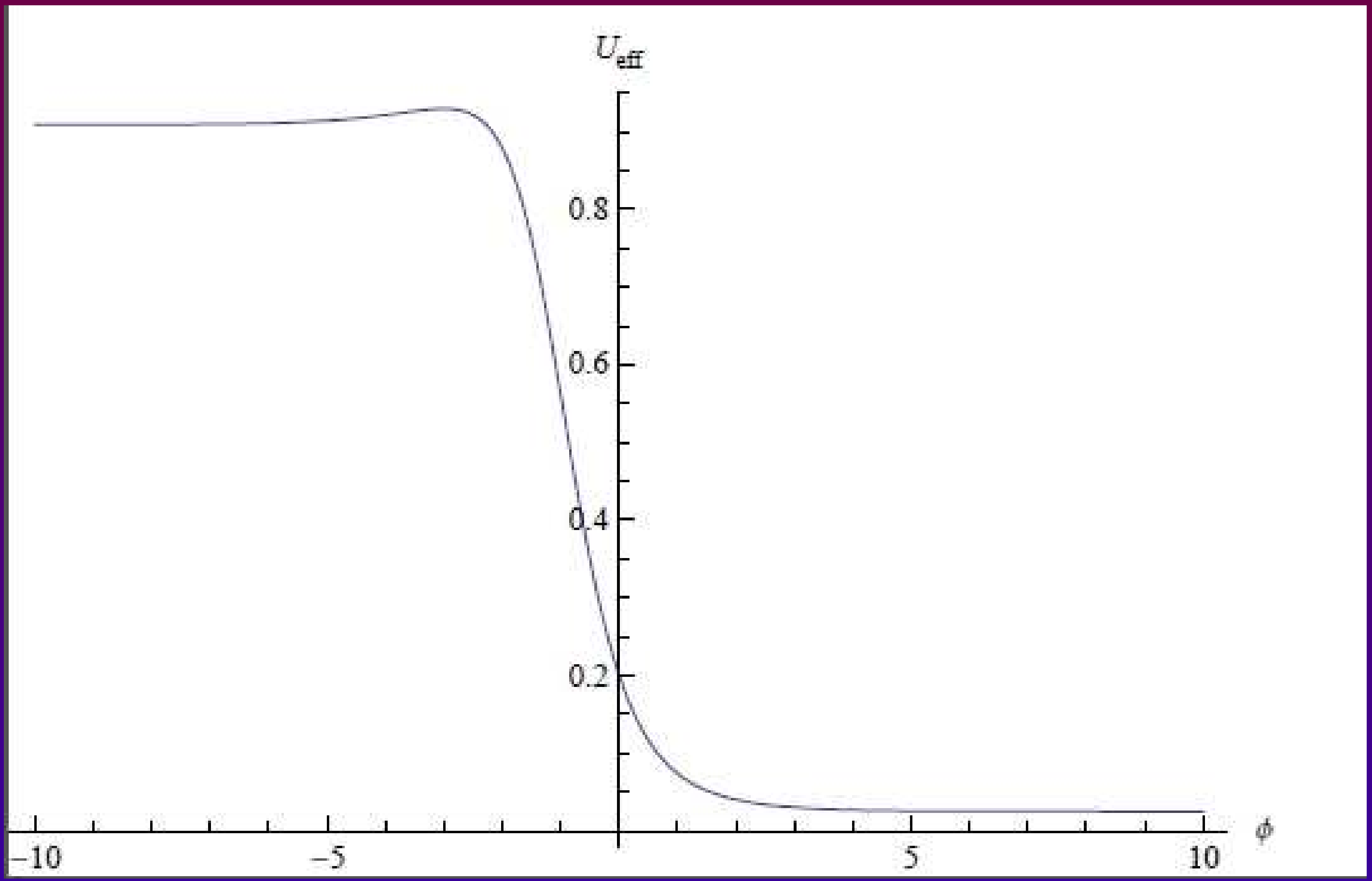
$U_{\mathrm{eff}}(\varphi,\sigma)$ has few remarkable properties.

First, $U_{\mathrm{eff}}(\varphi,\sigma)$ possesses two infinitely large flat regions (when σ is fixed):

(a) $(-)$ flat region for large negative values of the “inflaton” φ;

(b) $(+)$ flat region and large positive values of φ,

respectively, as depicted in Fig.1 on the next slide.



Qualitative shape of the effective scalar potential U_{eff} (26) as function of φ for $M_1 > 0$.

- In the $(+)$ flat region (large positive “inflaton” values) (26) reduces to:

$$U_{\text{eff}}\left(\varphi,\sigma\right) \simeq U_{(+)}(\sigma) = \frac{\left(\frac{\lambda}{4}\left((\sigma_a)^*\sigma_a - \mu^2\right)^2 + f_1\right)^2}{4\chi_2 f_2} \; . \qquad (27)$$

- (27) yields as a lowest lying vacuum the Higgs one:

$$|\sigma| = \mu \; , \qquad (28)$$

i.e., we obtain the standard spontaneous breakdown of $SU(2)\times U(1)$ gauge symmetry.

- At the Higgs vacuum (28) we get from (27) a dynamically generated cosmological constant $\Lambda_{(+)}$:

$$U_{(+)}(\mu) \equiv 2\Lambda_{(+)} = \frac{f_1^2}{4\chi_2 f_2} \; . \qquad (29)$$

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- If we identify the integration constants in (26) with the fundamental constants of Nature – M_{Pl} (Planck mass) and M_{EW} (electro-weak mass scale) as $f_1 \sim M_{EW}^4$, $f_2 \sim M_{Pl}^4$, we are then naturally led to a very small vacuum energy density:

$$U_{(+)}(\mu) \sim M_{EW}^8/M_{Pl}^4 \sim 10^{-122} M_{Pl}^4 \,, \qquad (30)$$

which is the right order of magnitude for the present epoch’s vacuum energy density. Therefore, we can identify the $(+)$ flat region (large positive “inflaton” values) of $U_{\rm eff}$ (26) as describing the present “late” universe.

- In the $(-)$ flat region (large negative "inflaton" values) (26) reduces to:

$$U_{\rm eff}(\varphi,\sigma) \simeq U_{(-)} \equiv \frac{M_1^2}{4(\chi_2\, M_2 - 2M_0)} \; . \qquad (31)$$

If we take $M_1 \sim M_2 \sim 10^{-8} M_{Pl}^4$ and $M_0 \sim M_{EW}^4$, then the vacuum energy density $U_{(-)}$ (31) becomes $U_{(-)} \sim 10^{-8} M_{Pl}^4$, which conforms to the Planck Collaboration data for the energy scale of inflation (of order $10^{-2} M_{Pl}$). This allows to identify the $(-)$ flat region (large negative "inflaton" values) of the "inflaton+Higgs" effective potential (26) as describing the "early" universe, in particular, the inflationary epoch.

- In the $(-)$ flat region the effective potential (31) is σ-field independent. Thus, the Higgs-like iso-doublet scalar field σ_a remains *massless* in the "early" (inflationary) Universe and accordingly there is **no electro-weak spontaneous symmetry breaking** there.

In a remarkable paper from 1986 J. Bekenstein proposed the intriguing idea about a gravity-assisted spontaneous symmetry breaking of electro-weak (Higgs) type without invoking unnatural (according to BekensteinŠs opinion)ingredients like negative mass squared and a quartic self-interaction for the Higgs field. To implement this idea, consider a small modification of (12):

$$\widehat{S} = \int d^4x\, \Phi(A)\Big[g^{\mu\nu} R_{\mu\nu}(\Gamma) - 2\Lambda_0 \frac{\Phi(A)}{\sqrt{-g}} + X + \widehat{f}_1 e^{\alpha\varphi} + X_\sigma - V_0(\sigma) e^{\alpha\varphi}\Big] + \int d^4x\, \Phi(B)\Big[U(\varphi) + L_3(\mathcal{A},\mathcal{B}) + \frac{\Phi(H)}{\sqrt{-g}}\Big] , \tag{32}$$

where in this case the bare Higgs-like pontial is the standard mass term:

$$V_0(\sigma) = m_0^2\,(\sigma_a)^*(\sigma_a) \; . \tag{33}$$

Also here we have an additional term qudratic w.r.t. $\Phi(A)$ with a small parameter Λ_0 later to be identified with the present (“late” Universe) CC.

Upon passage from (32) to the physical Einstein frame the "inflaton+Higgs" effective potential becomes:

$$U_{\rm eff}\left(\varphi,\sigma\right) = \frac{\Big[\Big(-\widehat{f}_1 + m_0^2\,(\sigma_a)^*(\sigma_a)\Big)e^{\alpha\varphi} + M_1\Big]^2}{4\big[\chi_2(f_2 e^{2\alpha\varphi} + M_2) - 2M_0\big]} + 2\Lambda_0\;. \qquad (34)$$

In the $(+)$ flat region (34) reduces to:

$$U_{\rm eff}\left(\varphi,\sigma\right) \simeq U_{(+)}(\sigma) = \frac{\Big(-\widehat{f}_1 + m_0^2\,(\sigma_a)^*(\sigma_a)\Big)^2}{4\chi_2\, f_2} + 2\Lambda_0\;. \qquad (35)$$

Spontaneous EW symmetry breaking occurs at $|\sigma_{\rm vac}| = \frac{1}{m_0}\sqrt{\widehat{f}_1}$ with natural identification of orders of magnitude $\widehat{f}_1 \sim M_{EW}^4\;,\; m_0 \sim M_{EW}$. Thus, the residual cosmological constant Λ_0 has to be identified with the current epoch observable cosmological constant ($\sim 10^{-122} M_{Pl}^4$).

Quintessence Stabilized via Gauss-Bonnet/Inflaton Coupling

Stability Issues: It is desirable that the “late” Universe epoch, instead of the infinitely large $(+)$ flat region, would be described in terms of a stable minimum of the effective “inflaton” potential. To this end we will introduce an additional *linear* coupling of the “inflaton” to Gauss-Bonnet gravitational term (for simplicity we discard here the “darkon” field):

$$S_{\mathrm{EF+GB}} = \int d^4x \sqrt{-\bar{g}} \Big[R(\bar{g}) + \bar{X} + \bar{X}_\sigma - U_{\mathrm{eff}}\big(\varphi,\sigma\big) - \frac{\chi_2}{4g^2}\bar{F}^2(\mathcal{A}) - \frac{\chi_2}{4g'^{\,2}}\bar{F}^2(\mathcal{B}) - b\,\varphi\,\bar{\mathcal{R}}_{\mathrm{GB}} \Big] \, , \qquad (36)$$

with $U_{\mathrm{eff}}\big(\varphi,\sigma\big)$ as in (26), and:

$$\bar{\mathcal{R}}_{\mathrm{GB}} = \bar{R}_{\mu\nu\kappa\lambda}\bar{R}^{\mu\nu\kappa\lambda} - 4\bar{R}_{\mu\nu}\bar{R}^{\mu\nu} + \bar{R}^2 \, , \qquad (37)$$

where all objects with superimposed bars are defined w.r.t. second-order formalism with the Einstein-frame metric $\bar{g}_{\mu\nu}$.

Here we will be interested in “vacuum” solutions, *i.e.*, for constant values of the matter fields. The corresponding equations of motion for constant φ and σ read:

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = -\frac{1}{2}\bar{g}_{\mu\nu}U_{\rm eff}(\varphi,\sigma) \ , \quad (38)$$

$$\frac{\partial}{\partial\varphi}U_{\rm eff}(\varphi,\sigma) + b\mathcal{R}_{\rm GB} = 0 \ ; \quad (39)$$

$$\frac{\partial}{\partial\sigma_a}U_{\rm eff}(\varphi,\sigma) = 0 \quad \longrightarrow \quad \frac{\partial}{\partial\sigma_a}V_0(\sigma) = 0$$

$$\longrightarrow \quad (\sigma_a)^*\left((\sigma_{a'})^*\sigma_{a'} - \mu^2\right) = 0 \quad \longrightarrow \quad |\sigma_{\rm vac}| = \mu \ \ or \ \ |\sigma_{\rm vac}| = 0 \ . \quad (40)$$

For constant φ and σ the solution to (38) is maximally symmetric:

$$\bar{R}_{\mu\nu\kappa\lambda} = \frac{1}{6}U_{\rm eff}(\varphi,\sigma)\left(\bar{g}_{\mu\kappa}\bar{g}_{\nu\lambda} - \bar{g}_{\mu\lambda}\bar{g}_{\nu\kappa}\right) \ , \quad (41)$$

which yields for the Gauss-Bonnet term (37):

$\mathcal{R}_{\rm GB} = \frac{2}{3}\Big(U_{\rm eff}(\varphi,\sigma)\Big)^2$, and insert it in (39):

$$\frac{\partial}{\partial\varphi}U_{\rm eff}(\varphi,\sigma_{\rm vac}) + \frac{2b}{3}\Big(U_{\rm eff}(\varphi,\sigma_{\rm vac})\Big)^2 = 0 \; , \qquad (42)$$

with $\sigma_{\rm vac}$ as in (40). Eq.(42) implies that in fact the total effective inflaton potential after introducing Gauss-Bonnet/inflaton linear coupling is modified from $U_{\rm eff}(\varphi,\sigma_{\rm vac})$ (26) to the following one:

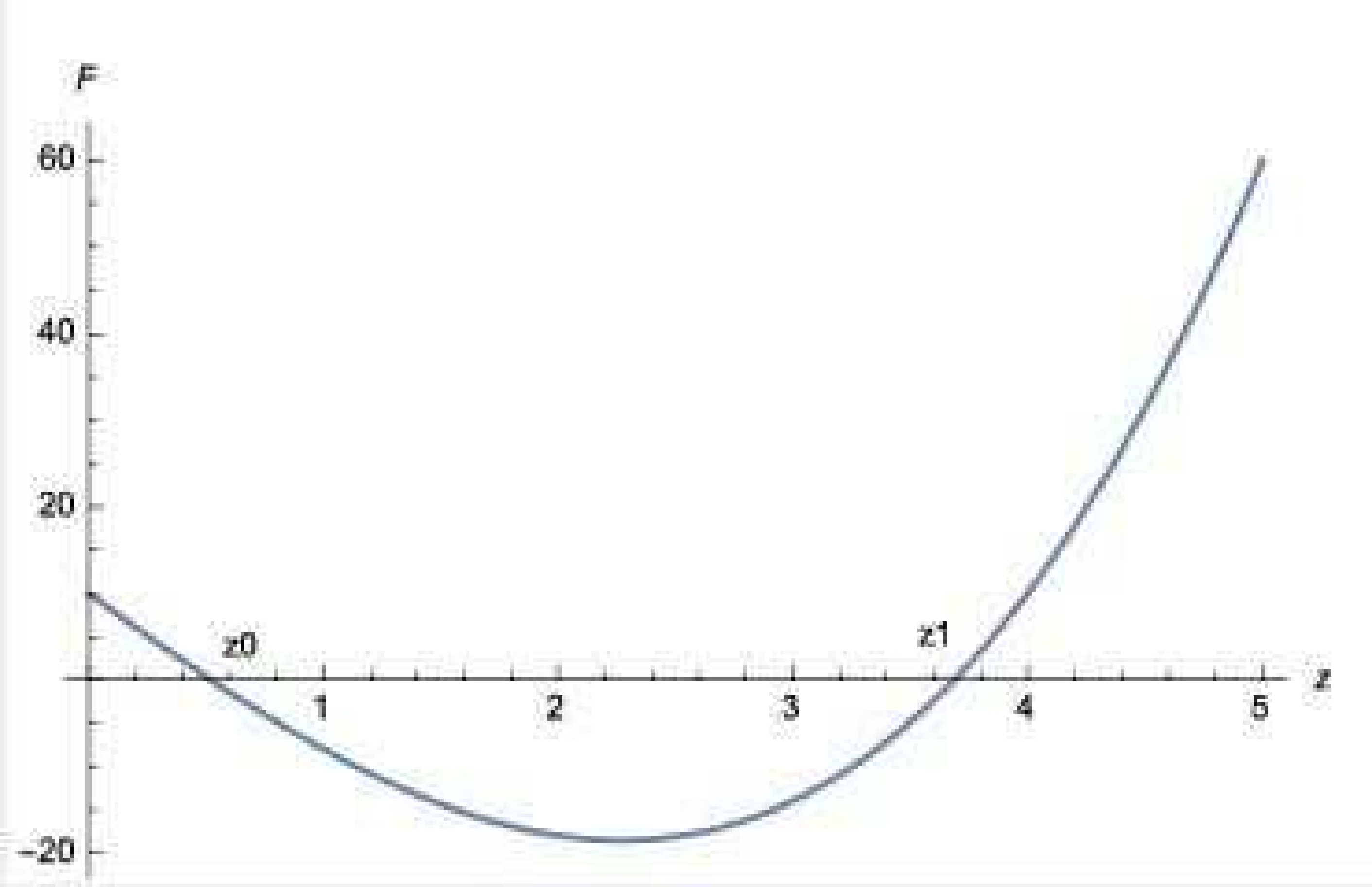
$$V_{\rm total}(\varphi,\sigma_{\rm vac}) = U_{\rm eff}(\varphi,\sigma_{\rm vac}) + \frac{2b}{3}\int^{\varphi} d\phi\Big(U_{\rm eff}(\phi,\sigma_{\rm vac})\Big)^2 \; , \qquad (43)$$

$$\frac{\partial}{\partial\varphi}V_{\rm total}(\varphi,\sigma_{\rm vac}) = \frac{b\,M_1^4\big(e^{-\alpha\varphi}+\widetilde{f}_1/M_1\big)}{24\chi_2^2 M_2^2\big(e^{-2\alpha\varphi}+f_2/M_2\big)}F(e^{-\alpha\varphi}) = 0 \; , \qquad (44)$$

$$F(z) \equiv z^3 + \frac{3\widetilde{f}_1}{M_1}\Big(1+\frac{4\alpha\chi_2 M_2}{b\,M_1^2}\Big)z^2 - \frac{3\widetilde{f}_1^2}{M_1^2}\Big(\frac{4\alpha\chi_2 f_2}{b\,\widetilde{f}_1^2}-1\Big)z + \frac{\widetilde{f}_1^3}{M_1^3} = 0 \; , \qquad (45)$$

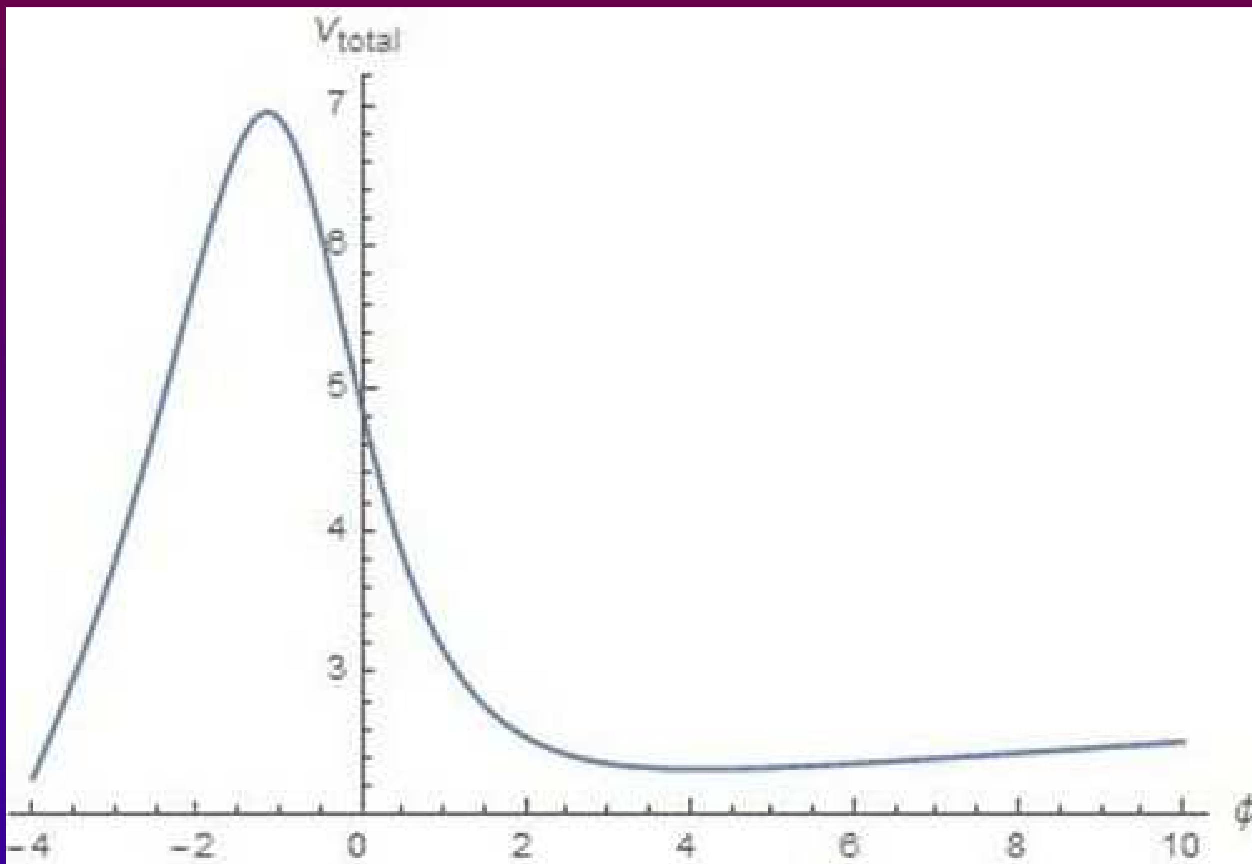
where $\widetilde{f}_1 \equiv f_1 + \frac{\lambda}{4}(|\sigma_{\rm vac}|^2-\mu^2)^2$. Therefore, the “vacuum” solutions $z \equiv e^{-\alpha\varphi_{\rm vac}}$ must be real positive roots of the cubic polynomial $F(z)$.

Qualitative plot of the cubic polynomial $F(z)$ (45):



The root z_0 correspond to a stable minimum of total effective inflaton potential $V_{\text{total}}(\varphi, \sigma_{\text{vac}})$ (43), whereas the root z_1 corresponds to a maximum of $V_{\text{total}}(\varphi, \sigma_{\text{vac}})$.

Qualitative shape of the total effective “inflaton” potential $V_{\text{total}}(\varphi, \sigma_{\text{vac}})$ (43) as function of φ after adding inflaton coupling to Gauss-Bonnet term.



(i) Minimum – “late” Universe evolution

(ii) Maximum (sufficiently smooth for small GB coupling b) – start of “hill-top” inflation [Hawking-Hertog, 2002]

Conclusions

Employing non-Riemannian spacetime volume-forms (volume elements) in generalized gravity-matter theories allows for several interesting developments:

- Simple unified description of dark energy and dark matter as manifestation of the dynamics of a single non-canonical scalar field (“darkon”).
- Construction of a new class of models of gravity interacting with a scalar “inflaton” φ, as well as with other phenomenologically relevant matter including Higgs-like scalar σ, which produce an effective full scalar potential of φ, σ with few remarkable properties.

Conclusions

- The “inflaton” effective potential (at fixed σ) possesses two infinitely large flat regions with vastly different energy scales for large negative and large positive values of φ. This allows for a unified description of both “early” universe inflation as well as of present “dark energy” epoch in universe’s evolution.
- In the “early” universe the would-be Higgs field σ remains massless and decouples from the “inflaton” φ. The “early” universe evolution is described entirely in terms of the “inflaton” dynamics.
- In the post-inflationary epoch φ and σ exchange roles. The inflaton φ becomes massless and decoupled, whereas σ becomes a genuine Higgs field with a dynamically generated electro-weak-type symmetry breaking effective potential.

Conclusions

- A natural choice for the parameters involved conforms to quintessential cosmological and electro-weak phenomenologies.
- Linear “inflaton” coupling to the gravitational Gauss-Bonnet term stabilizes the “late” universe dynamics by creating a shallow minimum of the effective “inflaton” potential instead of the infinitely large $(+)$ flat region. Together with this the Gauss-Bonnet coupling converts the $(-)$ flat region of the effective “inflaton” potential into a local smooth maximum where the Hawking-Hertog “hill-top” inflationary mechanism is applicable.

Conclusions

THANK YOU – Merci beaucoup

Vielen Dank – Tack – Multumesc

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хвала – hvala – спасибо – благодаря
спасибі – дзякуй – dziękuję – təşəkkürlər

ありがとう – **شكرا** – **תודה לך** – **ευχαριστώ**

(Arigatō) (shukran) (touda lekha) (evharisto)

Modified-Measure Theories

Spacetime volume-forms (generally-covariant integration measures) are given by nonsingular maximal rank diff. forms ω:

$$\int_{\mathcal{M}} \omega(\ldots) = \int_{\mathcal{M}} dx^D \, \Omega(\ldots) \;\; , \;\; \omega = \frac{1}{D!} \omega_{\mu_1 \ldots \mu_D} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_D} \; , \quad (1)$$

$$\omega_{\mu_1 \ldots \mu_D} = -\varepsilon_{\mu_1 \ldots \mu_D} \Omega \;\; , \;\; dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_D} = \varepsilon^{\mu_1 \ldots \mu_D} \, dx^D \; . \quad (2)$$

The integration measure density Ω transforms as scalar density under general coordinate reparametrizations.

In standard generally-covariant theories (with action $S = \int d^D x \sqrt{-g} \mathcal{L}$) the Riemannian spacetime volume-form is defined through the “D-bein” (frame-bundle) canonical one-forms $e^A = e^A_\mu dx^\mu$ ($A = 0, \ldots, D-1$):

$$\omega = e^0 \wedge \ldots \wedge e^{D-1} = \det \| e^A_\mu \| \, d^D x = \sqrt{-\det \| g_{\mu\nu} \|} \, d^D x \; . \quad (3)$$

Modified-Measure Theories

There is NO *a priori* any obstacle to employ instead of $\sqrt{-g}$ another alternative **non-Riemannian** volume element as in (1)-(2) given by an **exact** D-form $\omega = dA$ where:

$$A = \frac{1}{(D-1)!} A_{\mu_1 \ldots \mu_{D-1}} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu-1} \;, \tag{4}$$

so that the **non-Riemannian** integration measure density reads:

$$\Omega \equiv \Phi(A) = \frac{1}{(D-1)!} \varepsilon^{\mu_1 \ldots \mu_D} \, \partial_{\mu_1} A_{\mu_2 \ldots \mu_D} \;. \tag{5}$$

Here $A_{\mu_1 \ldots \mu_{D-1}}$ is an auxiliary rank $(D-1)$ antisymmetric tensor gauge field, which will turn out to be (almost) pure-gauge degree of freedom. $\Phi(A)$, which is in fact the dual of the rank D field strength $F_{\mu_1 \ldots \mu_D} = \frac{1}{(D-1)!} \partial_{[\mu_1} A_{\mu_2 \ldots \mu_D]} = -\varepsilon_{\mu_1 \ldots \mu_D} \Phi(A)$, similarly transforms as scalar density under general coordinate reparametrizations.

yields for the slow-roll parameter ε (43):

$$\varepsilon \simeq \frac{4\alpha^2 M_1^2 e^{2\alpha\varphi}}{f_1^2} \ll 1 \quad \text{for large negative } \varphi.$$

Similarly, for the second slow-roll parameter we have:

$$\left| 2\, \frac{U''_{\text{eff}}}{U_{\text{eff}}} \right| \simeq \frac{4\alpha^2 |M_1| e^{\alpha\varphi}}{f_1} \ll 1 \quad \text{for large negative } \varphi.$$

The value of φ at the end of the slow-roll regime φ_{end} is determined from the condition $\varepsilon \simeq 1$ which through (45) yields:

$$e^{-2\alpha\varphi_{\text{end}}} \simeq \frac{4\alpha^2 M_1^2}{f_1^2} \tag{47}$$

The number of *e-foldings* N between two values of cosmological times t_* and t_{end} or analogously between two different values φ_* and φ_{end} becomes:

$$N = \int_{t_*}^{t_{\rm end}} H\,dt = \int_{\varphi_*}^{\varphi_{\rm end}} \frac{H}{\dot{\varphi}} d\varphi \simeq -\int_{\varphi_*}^{\varphi_{\rm end}} \frac{3H^2}{U'_{\rm eff}} d\varphi \simeq -\int_{\varphi_*}^{\varphi_{\rm end}} \frac{U_{\rm eff}}{2U'_{\rm eff}} d\varphi,$$

$$N \simeq \frac{f_1}{4\alpha^2 M_1}\left(e^{-\alpha\varphi_*} - e^{-\alpha\varphi_{\rm end}}\right).$$

the subscript $*$ is used to indicate the epoch where the cosmological scale exits the horizon.

the scalar spectral index n_s can be expresses in terms of the number of e-foldings N to give:

$$n_s \simeq 1 - \alpha\,[1 + 2\alpha\, N]^{-1} - \left[\frac{1}{4\pi^2} + 1\right]\,[1 + 2\alpha\, N]^{-2}.$$

the relation between the tensor-to-scalar ratio r and the spectral index n_s , i.e., the consistency relation, $n_s = n_s(r)$, is given by:

$$n_s \simeq 1 - \frac{\pi\,\alpha}{\sqrt{2}}\, r^{1/2} - \frac{\pi^2}{2}\left[\frac{1}{4\pi^2} + 1\right] r\,.$$