INTRISTIC NON-COMMUTATIVITY OF CLOSED STRING THEORY (QUANTUM GRAVITY)

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ETC.

EFFECTIVE PICTURE OF STRINGS:

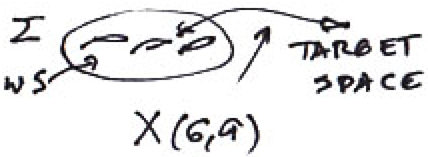
"TARGET SPACE = INDEX SPACE"

• OLD PICTURE:

$\xrightarrow{L \gg \lambda}$ $\cdot$

INDEX SPACE

$\Phi(x)$ — LOCAL FIELD



(EFFECTIVE FIELD THEORY / WILSONIAN RG

PARTICLE QUANTA

DERIVATIVE EXPANSION

DECOUPLING OF IR & UV)

..

• NEW PICTURE:

$[x, \tilde{x}] = i\lambda^2 (2\pi)$

$\xrightarrow{L \gg \lambda}$

$\Phi(x, \tilde{x})$ — NON-COMMUTATIVE FIELD THEORY

$(\hbar = \lambda\epsilon$

$\alpha' = \frac{\lambda}{\epsilon})$

(VIOLATION OF DECOUPLING

UV/IR MIXING

DOUBLE SCALE RG; SELF-DUAL FIXED POINT)

"TARGET SPACE ≠ INDEX SPACE"

CONSIDER

FLAT (FREE) STRING: $S_P \sim \int {}^*dx^\mu \wedge dx^\nu \, \eta_{\mu\nu}$

$$\delta S_P = 0 \;\longrightarrow\; \Box X = 0 \;\rightarrow\; 2 \text{ SOLUTIONS}$$

$$\boxed{X(\tau,\sigma) = X_R(\tau+\sigma) + X_L(\tau-\sigma)} \qquad \lambda \equiv \sqrt{\frac{\pi\alpha'}{2}}$$

$$X_L(\tau-\sigma) \equiv x_L + \frac{\alpha'}{2} p_L (\tau-\sigma) + i\lambda \sum_{n=-\infty}^{+\infty} \frac{1}{n} \alpha_n e^{-in(\tau-\sigma)}$$

$$X_R(\tau+\sigma) \equiv x_R + \frac{\alpha'}{2} p_R (\tau+\sigma) + i\lambda \sum_{n=-\infty}^{+\infty} \frac{1}{n} \tilde{\alpha}_n e^{-in(\tau+\sigma)}$$

ALSO:

$$\boxed{\tilde{X}(\tau,\sigma) = X_R(\tau+\sigma) - X_L(\tau-\sigma)}$$

NOTE: $\underline{dX = {}^*d\tilde{X}}$, $\underline{\text{i.e.}}$ $\underline{\partial_\tau X = \partial_\sigma \tilde{X};\; \partial_\sigma X = \partial_\tau \tilde{X}}$

$\tilde{X} \rightarrow$ T-DUALITY $\quad \underline{\tilde{X}(\tau, \sigma+2\pi) = \tilde{X}(\tau,\sigma) + 2\pi\alpha' p} \quad \left(p = \frac{p_L + p_R}{2}\right)$

$(R; 0 \rightarrow 0; \tilde{R} \;;\; R\tilde{R} = 2\lambda^2) \qquad \left(2\pi\alpha' p = \int_0^{2\pi} d\sigma \, \partial_\tau X\right)$

USUALLY $[x, \tilde{x}] = 0$ (TEXTBOOK)

"ZERO MODES x_L, x_R COMMUTE"

$\boxed{\text{NOT TRUE!}}$

4 DIFFERENT CALCULATIONS TO SEE THAT $[x, \tilde{x}] = i\lambda^2 (2\pi)$

1) PERHAPS THE SIMPLEST: START FROM CANONICAL

$$[\hat{X}(\tau, \sigma_1), \partial_\tau \hat{X}(\tau, \sigma_2)] = 2\pi i \hbar \alpha' \delta(\sigma_{12})$$

HOWEVER, $\partial_\tau X = \partial_\sigma \tilde{X}$

$$[\hat{X}(\tau, \sigma_1), \partial_\sigma \hat{\tilde{X}}(\tau, \sigma_2)] = 2\pi i \hbar \alpha' \delta(\sigma_{12})$$

INTEGRATE OVER σ; INVOKE <u>WS</u> CAUSALITY (MUTUAL LOCALITY)

$$[\hat{X}(\tau, \sigma_1), \hat{\tilde{X}}(\tau, \sigma_2)] = 2\pi i \lambda^2 [\pi - \theta(\sigma_{12})]$$

STAIRCASE DISTRIBUTION

ZERO MODES <u>DO NOT</u> COMMUTE $[\hat{x}^a, \hat{\tilde{x}}_b] = 2\pi i \lambda^2 \delta^a_b$

$(x \equiv x_L + x_R;\ \tilde{x} \equiv x_L - x_R)$

HEISENBERG ALGEBRA

USE THE DOUBLE NOTATION:

$$\mathbb{X}^A(\tau,\sigma) \equiv \left(X^\mu(\tau,\sigma) , \tilde{X}_\mu(\tau,\sigma) \right)$$

$$\Rightarrow \quad \left[\hat{\mathbb{X}}^A(\tau,\sigma_1), \hat{\mathbb{X}}^B(\tau,\sigma_2) \right] = 2i\lambda^2 \left[\pi \omega^{AB} - \eta^{AB}\theta(\sigma_{12}) \right]$$

ZERO MODES

OSCILLATORS

$\omega^{AB} \rightarrow Sp(2D)$

↳ dim OF SPACETIME

$\eta^{AB} \rightarrow O(D,D)$

$\theta(\sigma)$ – STAIRCASE DISTRIBUTION

2) WHERE IS $\tilde{X}$ IN THE POLYAKOV ACTION?

T-DUALITY $\rightarrow$ $S_P \rightarrow S_P + \hbar \int \omega$

$$\omega \equiv \frac{1}{8\pi\lambda^2} \left[d\tilde{X}_a \wedge dX^a \right]$$

WZW-LIKE TERM

$\tilde{X}$ – TOPOLOGICAL DEGREE OF FREEDOM

(ONLY ZERO MODES DYNAMICAL)

3) SYMPLECTIC FORM FROM δS_P

$$\left(S_P \equiv \frac{1}{4\pi\alpha'} \int d^2\sigma \left[(\partial_\tau X)^2 - (\partial_\sigma X)^2 \right] \right)$$

BE CAREFUL WITH BOUNDARY TERMS

FOR A CYLINDRICAL WS CUT OPEN AT $\sigma = 0$

$$(\sigma \in [0, 2\pi], \quad \tau \in [\tau_0, \tau_1])$$

USE $\partial_\tau X = \partial_\sigma \tilde{X}$ ON THE BOUNDARY

$\delta S_P \rightarrow$ SYMPLECTIC 1-FORM $\Theta(\tau)$

$\delta \Theta = \Omega$ - SYMPLECTIC 2-FORM

$$\Omega = \delta p \wedge \delta x + \delta \tilde{p} \wedge (\delta \tilde{x} - \pi \alpha' \delta p) + \text{OSCILLATOR TERMS}$$

BERRY-PHASE-LIKE TERM $-\pi\alpha' \delta\tilde{p} \wedge \delta p$

$$\Rightarrow [\hat{x}^a , \hat{\tilde{x}}_b] = 2\pi i \lambda^2 \delta^a_b$$ + CANONICAL COMMUTATORS

4) COCYCLES OF TEXTBOOK VERTEX OPERATORS:

$$\hat{U}_k = e^{i k_L x_L + i k_R x_R}$$ (COMMUTING WITH ZERO MODES!)

$$\hat{U}_k \hat{U}_{k'} = \epsilon_{k,k'} \hat{U}_{k+k'}$$

DEDUCE (FROM MUTUAL LOCALITY) $\epsilon_{k,k'} = e^{2\pi i \lambda^2 \tilde{k} k'}$

REALIZE INTRINSIC NONCOMMUTATIVITY $[x, \tilde{x}] = 2\pi i \lambda^2$

NOW $\hat{U}_k = e^{i k \hat{x}} \, e^{i \tilde{k} \hat{\tilde{x}}}$ $\quad (x = x_L + x_R)$ $\quad (\tilde{x} = x_L - x_R)$

$\downarrow$

REPS OF HEISENBERG-WEYL

COCYCLES $\to$ 1 !

(NEW INSIGHT ON ASYMMETRIC ORBIFOLDS & OTHER NON-GEOMETRIC BACKGROUNDS)

TURN ON G_{ab} & B_{ab} :

$$S_{P(G,B)} = \frac{1}{4\pi\alpha'}\int d^2\sigma \left(G_{ab}\left[\partial_\tau X^a \partial_\tau X^b - \partial_\sigma X^a \partial_\sigma X^b\right] + 2B_{ab}\,\partial_\tau X^a \partial_\sigma X^b \right)$$

$$\Rightarrow \quad \underline{[\hat{x}^a, \hat{x}^b] = 0}\,, \quad \underline{[\hat{x}^a, \hat{\tilde{x}}_b] = 2\pi i \lambda^2 \delta^a{}_b}\,, \quad \underline{[\hat{\tilde{x}}_a, \hat{\tilde{x}}_b] = -4\pi i \lambda^2 B_{ab}}$$

INTRODUCE

$$\eta_{AB} = \begin{pmatrix} 0 & \delta_a{}^b \\ \delta^a{}_b & 0 \end{pmatrix} \qquad \omega_{AB} = \begin{pmatrix} 0 & -\delta_a{}^b \\ \delta^a{}_b & 0 \end{pmatrix} \qquad K = \eta^{-1}\omega$$

$O(D,D)$ $\qquad$ $Sp(2D)$ $\qquad$ $K^2 = 1$

IF $B_{ab} = 0$

$$\boxed{\Omega = \eta_{AB}\, \delta \mathbb{P}^A \wedge \delta \mathbb{X}^B + \frac{\pi\alpha'}{2}\, \omega_{AB}\, \delta \mathbb{P}^A \wedge \delta \mathbb{P}^B}$$

IF $B_{ab} \neq 0$

$$\eta^{(B)}_{AB} = \eta_{AB} = \begin{pmatrix} 0 & \delta_a{}^b \\ \delta^a{}_b & 0 \end{pmatrix}, \qquad \omega^{(B)}_{AB} = \begin{pmatrix} -2B_{ab} & -\delta_a{}^b \\ \delta_a{}^b & 0 \end{pmatrix}$$

$$\boxed{\Omega = \eta_{AB}\, \delta \mathbb{P}^A \wedge \delta \mathbb{X}^B + \frac{\pi\alpha'}{2}\, \omega^{(B)}_{AB}\, \delta \mathbb{P}^A \wedge \delta \mathbb{P}^B}$$

THEN $\quad \underline{[\mathbb{X}^A, \mathbb{X}^B] = 2\pi i \lambda^2 (\omega^{(B)})^{AB}}\,, \qquad (\omega^{(B)})^{AB} = \begin{pmatrix} 0 & \delta^a{}_b \\ -\delta_a{}^b & -2B_{ab} \end{pmatrix}$

NOTE: B-FIELD TRANSFORMATION

$$\mathbb{X} \equiv (x^a, \tilde{x}_a) \xmapsto{B} (x^a, \tilde{x}_a + B_{ab}x^b)$$

WHICH LEADS TO $\underbrace{[\hat{x}^a, \hat{x}^b] = 0}$, $\underbrace{[\hat{x}^a, \hat{\tilde{x}}_b] = 2\pi i \lambda^2 \delta^a_b}$, $\underbrace{[\hat{\tilde{x}}_a, \hat{\tilde{x}}_b] = -4\pi i \lambda^2 B_{ab}}$

HOWEVER, WE CAN ALSO CONSIDER

$$(x^a, \tilde{x}_a) \xmapsto{\beta} (x^a + \beta^{ab}\tilde{x}_b, \tilde{x}_a)$$

THIS LEADS TO NON-COMMUTATIVITY OF x^a! (STRINGY NON-LOCALITY)

$\underbrace{[\hat{x}^a, \hat{x}^b] = 4\pi i \lambda^2 \beta^{ab}}$, $\underbrace{[\hat{x}^a, \hat{\tilde{x}}_b] = 2\pi i \lambda^2 \delta^a_b}$, $\underbrace{[\hat{\tilde{x}}_a, \hat{\tilde{x}}_b] = 0}$

FINALLY, NOTE THAT FOR THE B-FIELD BACKGROUND

$$\underline{[\hat{\tilde{x}}_a, [\hat{\tilde{x}}_b, \hat{\tilde{x}}_c]] + \text{cyclic} = H_{abc}(x)}$$

3-COCYCLES

$$H_{abc} = \partial_a B_{bc} + \text{cyclic} \longrightarrow H\text{-FLUX}$$

POSSIBILITY OF NON-ASSOCIATIVITY.

ALL FLUXES $(H, \underbrace{F, R}_{\text{DOUBLE FIELD THEORY}} \ldots)$ FROM NON-COMMUTATIVITY & NON-ASSOCIATIVITY OF "PHASE SPACE" $\mathbb{X}$

(T-DUALITY)

COVARIANT FORMULATION (METASTRING) FLM

"PHASE SPACE" $\longleftrightarrow$ $\mathbb{X}^A = \begin{pmatrix} x^a/\lambda \\ \tilde{x}_a/\epsilon \end{pmatrix}$

$$S_{MS} = \frac{1}{4\pi}\int \left[\partial_\tau \mathbb{X}^A (\eta_{AB} + \omega_{AB})(\mathbb{X})\, \partial_\sigma \mathbb{X}^B - \partial_\sigma \mathbb{X}^A H_{AB}(\mathbb{X})\, \partial_\sigma \mathbb{X}^B\right]$$

$(\hbar = \lambda\epsilon,\ \alpha' = \frac{\lambda}{\epsilon})$

$\eta \to O(D,D) \qquad \omega \to Sp(2D) \qquad H \to O(2,2(D-1))\left[\begin{pmatrix} G_{ab} & 0 \\ 0 & G^{-1}_{ab} \end{pmatrix}\right]$

THEN IMMEDIATELY:

$$[\hat{\mathbb{X}}^A, \hat{\mathbb{X}}^B] = 2i\left[\pi\omega^{AB} - \eta^{AB}\,\Theta(\sigma,\tau)\right]$$

$\Theta(\sigma,\tau) \to$ STAIRCASE

NOTE $J \equiv \eta^{-1}H, \quad J^2 = 1$

$(\mathbb{X}^A(\sigma + 2\pi) = \mathbb{X}^A(\sigma) + \Delta^A)$

GENERAL MONODROMIES

T-DUALITY $\quad \mathbb{X} \to J(\mathbb{X})$

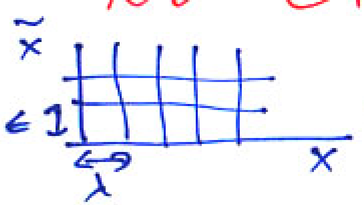
CHIRAL FORMULATION (AND NON-LOCAL) $\longleftrightarrow$ "FULLY COMPACTIFIED" $\leftarrow$ MONSTER, BORCHERDS

D = 26 BOSONIC STRING $\longleftrightarrow$ (UNIQUE SELF-DUAL LORENTZIAN LATTICE)

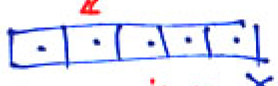
No CTCs $\to$ "QUANTUM MODULAR SPACE TIME"

$\Downarrow$

MODULAR VARIABLES OF QUANTUM THEORY



$\Rightarrow$



SPACE TIME "EXTENSIFICATION"

(SCHWINGER; AHARONOV)

MODULAR VARIABLES: FROM QUANTUM THEORY TO STRING THEORY

I) QUANTUM THEORY: NON-LOCALITY ⊕ CAUSALITY $R \updownarrow$

(AHARONOV)

INSTEAD OF $\hat{q}, \hat{p}$ WITH $[\hat{q}, \hat{p}] = i\hbar$ USE $[\hat{q}]_R \equiv \hat{q} \bmod (R)$

$[\hat{p}]_R \equiv \hat{p} \bmod \left(\frac{2\pi\hbar}{R}\right)$

(NON-LOCAL MODULAR VARIABLES $[\hat{q}]_R, [\hat{p}]_R$)

FULLY COVARIANT!

WITH $[[\hat{q}]_R, [\hat{p}]_R] = 0$!

$\Downarrow$

MODULAR SPACETIME

$\Downarrow$

II) (LOCAL) QUANTUM FIELD THEORY POSSIBLE BECAUSE OF CONSISTENCY BTW QUANTUM NON-LOCALITY & CAUSALITY

(STILL NEW INSIGHTS, AND PHYSICS! PROVIDED BY MODULAR VARIABLES)

"WEAK MEASUREMENTS" ETC

$\Uparrow$ $\Downarrow$

III) STRING THEORY IN METASTRING FORMULATION

- LIVES IN MODULAR SPACE TIME
- "GRAVITIZES" THE GEOMETRY OF QUANTUM THEORY IN MODULAR FORMULATION

$(\omega_{AB}(\mathbb{X}), \eta_{AB}(\mathbb{X}), H_{AB}(\mathbb{X}))$

- R is λ!

CONTEXTUAL

NOT-CONTEXTUAL

QUANTUM THEORY IN TERMS OF MODULAR VARIABLES:

$$\hat{q} \to \hat{U} = \exp\left(\frac{2\pi i}{\hbar}\hat{q}\hbar R^{-1}\right) \equiv \exp\left(2\pi i \frac{\hat{q}}{R}\right)$$

$$\hat{p} \to \hat{V} = \exp\left(\frac{i}{\hbar}\hat{p}R\right)$$

"VERTEX OPERATORS"

$$\left([\hat{q}, \hat{p}] = i\hbar \quad \Downarrow \right) \quad [\hat{U}, \hat{V}] = 0$$

USE $\hat{\mathbb{X}}^A = \begin{pmatrix} \hat{x}^a \\ \hat{\tilde{x}}_a \end{pmatrix}$ $\quad \hat{x}^a \equiv \frac{\hat{q}^a}{\lambda}$, $\hat{\tilde{x}}_a \equiv \frac{\hat{p}_a}{\epsilon}$ $\quad (\hbar = \lambda\epsilon)$

$$\Downarrow$$

$$\underline{W_{\mathbb{K}} \equiv \exp(2\pi i\, \omega(\mathbb{K}, \hat{\mathbb{X}}))} \quad ; \quad \underbrace{\omega(\mathbb{X}, \mathbb{Y}) = \hat{x}\cdot y - k\cdot\tilde{y}}_{Sp(2D)}$$

$\hookrightarrow$ COMMUTATIVE SUBALGEBRA OF HEISENBERG ALGEBRA:

MODULAR SPACETIME $\Leftarrow$ SELF-DUAL LATTICE Λ WRT ω! (NARAIN LATTICE)

$$\Lambda = \ell \oplus \bar{\ell} \longrightarrow \underbrace{\eta(\mathbb{P}, \mathbb{Q}) = \hat{p}\cdot q + p\cdot\hat{q}}_{O(D,D)}$$

FINALLY, VACUUM $\mathbb{P}^A|0\rangle = 0 \Rightarrow \underbrace{\hat{E}_H = H^{AB}\hat{\mathbb{P}}_A\hat{\mathbb{P}}_B}$, $\underline{\hat{E}_H|0\rangle = 0}$

$\uparrow$ TRANSLATION GENERATOR

$H^{AB} = H^{BA}$ $\quad O(2, 2(D-1))$ $\quad$ (RELATIVE π (OBSERVER DEPENDENT) LOCALITY)

NOTE: LORENTZ! $\quad \boxed{O(D-1,1) = \underset{\omega}{Sp(2D)} \cap \underset{\eta}{O(D,D)} \cap \underset{H}{O(2,2(D-1))}}$

EFFECTIVE FIELD DESCRIPTION OF CLOSED STRINGS

$$\Phi(x, \hat{x}) = \sum_{\omega} \phi_{\omega}(x)\, e^{i\omega \tilde{x}/\hat{\ell}}$$

$$[x, \hat{x}] = 2\pi i \lambda^2$$

THUS

$$\Phi(x, \tilde{x}) = \underset{\text{LOCAL EFFECTIVE FIELD}}{\underbrace{\Phi(x)}} + \overbrace{\sum_{\omega \neq 0} \phi_{\omega}(x)\, e^{i\omega \tilde{x}}}^{\text{NON-LOCAL FIELD}}$$

(DOUBLE RG
UV/IR MIXING)
SELF-DUAL FIXED POINTS

NON-PARTICLE QUANTA:

FROM NON-COMMUTATIVE CLOSED STRING ZERO MODES

$$S = \int \left[p\dot{x} + \hat{p}\dot{\hat{x}} - \alpha' \pi\, p \dot{\hat{p}} + N_1 \left(p^2 + \hat{p}^2 - M_1^2\right) + N_2\left(p\hat{p} - M_2^2\right)\right]$$

CONSTRAINTS $(p^2 + \hat{p}^2 = M_1^2)$ & $(p\hat{p} = M_2^2)$

FROM METASTRING CONSTRAINTS

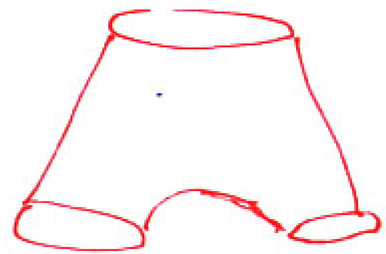
$$H: \partial_\sigma \mathbb{X}^A \partial_\sigma \mathbb{X}^B H_{AB} \approx 0$$

$$D: \partial_\sigma \mathbb{X}^A \partial_\sigma \mathbb{X}^B \eta_{AB} \approx 0$$

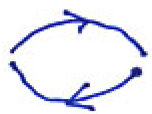
(USUALLY WE IGNORE $\tilde{x}$!!)
$(\tilde{x} \to 0)$
$(\hat{p} \to 0)$

INTERACTIONS:

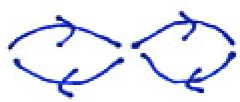
INCORPORATE
$[x, \tilde{x}] = 2\pi i \lambda^2$



$$\Phi_w(x) = \Phi^{(+)}(x, x+\pi w R)\, \Phi^{(-)}(x+\pi w R, x+2\pi w R)$$



CLOSED STRING ~ PRODUCT OF TWO OPEN STRINGS



$$(\Phi \circ \psi)^{(\pm)}_{w'}(x, x+\pi w R) \equiv \Phi^{(\pm)}(x, x+\pi w' R)\, \psi^{(\pm)}(x+\pi w' R, x+\pi w R)$$

REALIZATION OF THE HEISENBERG GROUP $[x, \tilde{x}] = 2\pi i \lambda^2$

NON-PERTURBATIVE FORMULATION $\partial_\sigma \hat{X}^A \to \frac{1}{\lambda}[\hat{X}^{26}, \hat{X}^A]$

INVOLVING $\hat{X}^A(\sigma, \tau) \to \hat{X}^a(\tau)$ $A = 0, 1 \ldots 25 \to a = 0, 1, \ldots, 25, 26$

$$S_{NP} \sim \frac{1}{\lambda}\int Tr\left(\partial_a \hat{X}^a [\hat{X}^b, \hat{X}^c]\, \eta_{abc}^{(*)} - \frac{1}{\lambda} H_{ac}^{(*)} [\hat{X}^a, \hat{X}^b][\hat{X}^c, \hat{X}^d] H_{bd}^{(*)} \right)$$

"27 = 11+16" ↳ SUSY, EMERGENT
↓
M-THEORY

11D — M-THEORY
COVARIANT MATRIX THEORY

$(\eta_{abc}^{(*)} \to \omega_{AB}^{(*)} + \eta_{AB}^{(*)})$

APPLICATIONS:

1) GENERIC "LOW ENERGY" PREDICTION OF STRING TH.

$\Phi(x, \hat{x})$ $[x, \hat{x}] = 2\pi i \lambda^2$

⇝ (NON-PARTICLE DARK MATTER WITH $\underbrace{\text{MILGROM'S SCALING}}_{}$) — $\sqrt{\lambda_{cc}}$ SENSITIVE

λ_{cc} – COSMOLOGICAL CONSTANT

D.M. WITH FRIENDS (EDMONDS, ~~M~~G, TAKEUCHI, FARRAH, HO)

2) NON-DECOUPLING ("MIXING") BTW UV & IR

HIERARCHY PROBLEM; NATURALNESS

3) VACUUM ENERGY λ_{cc} & T-DUALITY

(TSEYTLIN ⇝ FLM ; RELATED TO SEQUESTER OF KALOPER, PADILLA?)

$\int \sqrt{g \hat{g}} \, (R(g) + \hat{R}(\hat{g})) + \ldots$ → DFT

⋮

(BH INFORMATION PUZZLE)

⋮