Tachyon scalar field in DBI and RSII cosmological context

Neven Bilic, Goran S. Djordjevic, Milan Milosevic and Dragoljub D. Dimitrijevic

RBI, Zagreb, Croatia and FSM, University of Nis, Serbia

9th MATHEMATICAL PHYSICS MEETING:
School and Conference on Modern Mathematical Physics
18 - 23 September 2017, Belgrade, Serbia

Introduction

- The inflationary universe scenario in which the early universe undergoes a rapid expansion has been generally accepted as a solution to the horizon, flatness, etc. problems of the standard big-bang cosmology.
- Recent years - a lot of evidence from WMAP, Planck, etc. observations of the CMB.

Introduction

- We study (real) scalar field in cosmological context.

- General Lagrangian action:

$$S = \int d^4x \sqrt{-g}\,\mathcal{L}(X(\partial\phi),\phi)$$

- Lagrangian (Lagrangian density) of the standard form:

$$\mathcal{L}(\phi,\partial\phi) = X(\partial\phi) - V(\phi)$$

- Non-standard Lagrangian:

$$\mathcal{L}_{tach}(T,X) = -V(T)\sqrt{1+2X(\partial T)} \qquad X = \frac{1}{2} g^{\mu\nu}\partial_\mu T \partial_\nu T$$

Introduction

- The action:

$$S = \int d^4x \sqrt{-g}\, \mathcal{L}(X, T)$$

- In cosmology, scalar fields can be connected with a perfect fluid which describes (dominant) matter in the Universe.
- Components of the energy-momentum tensor:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$$

$$T_{\mu\nu} = (P + \rho) u_\mu u_\nu - P g_{\mu\nu}$$

Introduction

$$T_{\mu\nu} = (P + \rho) u_\mu u_\nu - P g_{\mu\nu}$$

- Pressure, matter density and velocity 4-vector, respectively:

$$P(X,T) \equiv \mathcal{L}(X,T)$$

$$\rho(X,T) \equiv 2X \frac{\partial \mathcal{L}}{\partial X} - \mathcal{L}(X,T)$$

$$u_\mu \equiv \frac{\partial_\mu T}{\sqrt{2X}}$$

Introduction

- Total action: term which describes gravity (Ricci scalar, Einstein-Hilbert action) plus term that describes cosmological fluid (scalar field Lagrangian):

$$S = \int d^4x \sqrt{-g}\,\left(R + \mathcal{L}(X,T)\right)$$

- Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu}$$

Tachyon field cosmology

- Tachyon lagrangian:

$$\mathcal{L}_{tach}(T,X) = -V(T)\sqrt{1+g^{\mu\nu}\partial_\mu T\partial_\nu T}$$

- EoM:

$$\left(g^{\mu\nu} - \frac{\partial^\mu T\partial^\nu T}{1-(\partial T)^2}\right)\partial_\mu T\partial_\nu T = -\frac{1}{V(T)}\frac{dV}{dT}(1-(\partial T)^2)$$

Tachyon field cosmology

- Tachyon lagrangian:

$$\mathcal{L}_{tach}(T,X) = -V(T)\sqrt{1+g^{\mu\nu}\partial_\mu T \partial_\nu T}$$

- Friedmann equations for spatially homogenous scalar field:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{Pl}^2}\frac{V}{(1-\dot{T}^2)^{1/2}}$$

$$\frac{\ddot{T}}{1-\dot{T}^2} + 3H\dot{T} + \frac{V'}{V} = 0$$

Tachyon field cosmology

- Rescaling:

$$x = \frac{T}{T_0} \qquad U(x) = \frac{1}{\lambda} V\left(\frac{T}{T_0}\right) = \frac{V(x)}{\lambda} \qquad \tau = \frac{t}{t_0}$$

- EoM:

$$\ddot{x} - 3HT_0\dot{x}^3 - \frac{U'(x)}{U(x)}\dot{x}^2 + 3HT_o\dot{x} + \frac{U'(x)}{U(x)} = 0$$

- Hubble parameter rescaling:

$$\tilde{H} = T_0 \cdot H$$

Tachyon field cosmology

- Dimensionless equations:

$$\tilde{H}^2 = \frac{X_0^2}{3}\frac{U(x)}{\sqrt{1-\dot{x}^2}}$$

$$\ddot{x} + X_0\sqrt{3U(x)(1-\dot{x}^2)^{3/2}}\dot{x} + \frac{(1-\dot{x}^2)}{U(x)}\frac{dU(x)}{dx} = 0$$

$$X_0 = \frac{\lambda T_0^2}{M_{Pl}^2}, \qquad \lambda = \frac{M_s^4}{g_s(2\pi)^3}$$

The Inflation

- Slow-roll regime, slow-roll parameters:

$$\epsilon_{i+1} \equiv \frac{d \ln |\, \epsilon_i \,|}{dN}, \quad i \geq 0, \quad \epsilon_0 \equiv \frac{H_*}{H}$$

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_2 = \frac{1}{H}\frac{\ddot{H}}{\dot{H}} + 2\epsilon_1$$

$$\epsilon_1 = \frac{3}{2}\dot{x}^2, \quad \epsilon_2 = 2\frac{\ddot{x}}{\tilde{H}\dot{x}}$$

- Number of e-folds:

$$N(t) = \int_{t_i}^{t_e} H(t)dt$$

The Inflation

- Number of e-folds:

$$N(x) = X_0^2 \int_{x_i}^{x_e} \frac{U(x)^2}{\mid U'(x) \mid} dx, \qquad \text{where } \varepsilon_1(x_e) = 1$$

- The scalar spectral index:

$$n_s = 1 - 2\epsilon_1(x_i) - \epsilon_2(x_i)$$

- The tensor-to-scalar ratio:

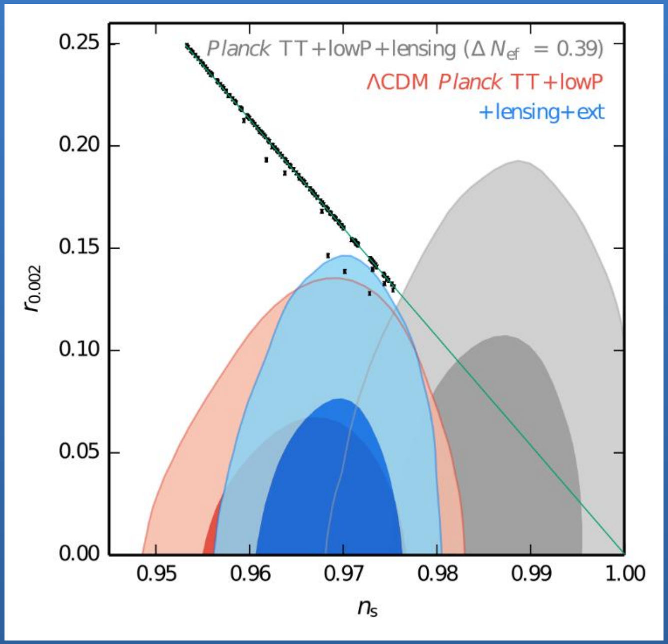
$$r = 16\epsilon_1(x_i)$$

The Inflation

- Numerical results:

$$60 \le N \le 120,\ 1 \le X_0 \le 12$$

$$U(x) = \frac{1}{\tilde{x}^4}$$

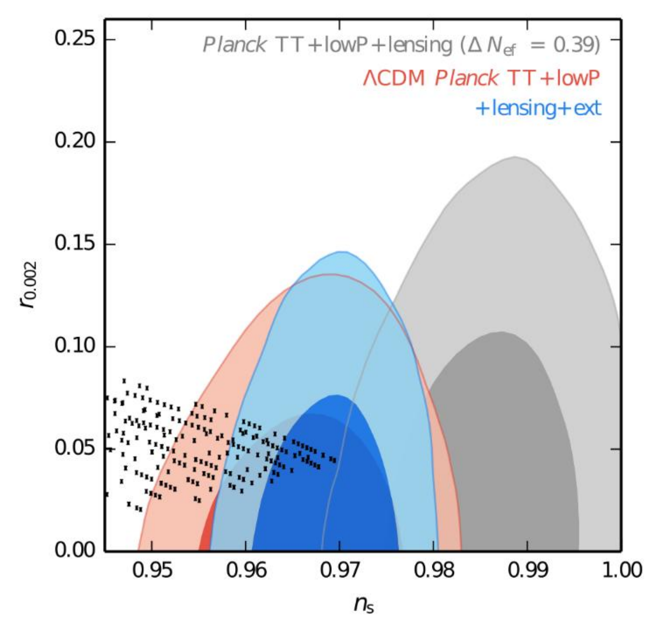


The Inflation

- Numerical results:

$$60 \le N \le 120,\ 1 \le X_0 \le 12$$

$$U(x) = \frac{1}{\cosh(\tilde{x})}$$



Tachyon inflation in an AdS braneworld

- Randall–Sundrum models (1999) imagine that the real world is a higher-dimensional universe described by warped geometry. More concretely, our universe is a five-dimensional anti-de Sitter space and the elementary particles except for the graviton are localized on a (3+1)-dimensional brane(s).
- A simple cosmological model of this kind is based on the RSII model .

Tachyon inflation in an AdS braneworld

- Cosmology on the brane is obtained by allowing the brane to move in the bulk. Equivalently, the brane is kept fixed at z=0 while making the metric in the bulk time dependent.
- The fluctuation of the interbrane distance implies the existence of the radion.
- Radion – a massless scalar field that causes a distortion of the bulk geometry.

Tachyon inflation in an AdS braneworld

- The bulk spacetime of the extended RSII model in Fefferman-Graham coordinates is described by the metric

$$ds^2_{(5)} = G_{ab} dX^a dX^b = \frac{1}{k^2 z^2}\left[\left(1 + k^2 z^2 \eta(x)\right) g^{\mu\nu} dx^\mu dx^\nu - \frac{1}{\left(1 + k^2 z^2 \eta(x)\right)^2} dz^2\right]$$

- Inverse of the AdS curvature radius – k
- Radion field – $\eta(x)$
- Fifth coordinate – z

Tachyon inflation in an AdS braneworld

- Add dynamical 3-brane, i.e. tachyon field (in terms of induced metric).
- The action, after integrating out fifth coordinate *z*:

$$S = \int d^4x\sqrt{-g}\left(-\frac{R}{16\pi G} + \frac{1}{2}g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu}\right) - \int d^4x\sqrt{-g}\,\frac{\sigma}{k^4\Theta^4}(1+k^2\Theta^2\eta)^2\sqrt{1-\frac{g^{\mu\nu}\Theta_{,\mu}\Theta_{,\nu}}{(1+k^2\Theta^2\eta)^3}}$$

- Radion field (canonical) – Φ
- Tachyon field – Θ

$$\eta = \sinh^2\left(\sqrt{4/3\pi G}\,\Phi\right)$$

Tachyon inflation in an AdS braneworld

- In the absence of radion – tachyon condenzate:

$$S_{\mathrm{br}}^{(0)} = -\int d^4x\sqrt{-g}\,\frac{\lambda}{\Theta^4}\sqrt{1-g^{\mu\nu}\Theta_{,\mu}\Theta_{,\nu}}$$

- Going back, lagrangian we are playing with:

$$\mathcal{L} = \frac{1}{2}g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu} - \frac{\lambda\psi^2}{\Theta^4}\sqrt{1-\frac{g^{\mu\nu}\Theta_{,\mu}\Theta_{,\nu}}{\psi^3}}$$

$$\psi = 1 + k^2\Theta^2\eta$$

$$\lambda = \frac{\sigma}{k^4}$$

Tachyon inflation in an AdS braneworld

- Hubble expansion rate H in standard cosmology (without brane):

$$H \equiv \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}\mathcal{H}}$$

- Hubble expansion rate H in RS cosmology:

$$H \equiv \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}\mathcal{H}\left(1+\frac{2\pi G}{3k^2}\mathcal{H}\right)}$$

Tachyon inflation in an AdS braneworld

- Hamilton's equation:

$$\dot{\Phi} = \frac{\partial \mathcal{H}}{\partial \Pi_{\Phi}}$$

$$\dot{\Theta} = \frac{\partial \mathcal{H}}{\partial \Pi_{\Theta}}$$

$$\dot{\Pi}_{\Phi} + 3H\Pi_{\Phi} = -\frac{\partial \mathcal{H}}{\partial \Phi}$$

$$\dot{\Pi}_{\Theta} + 3H\Pi_{\Theta} = -\frac{\partial \mathcal{H}}{\partial \Theta}$$

Tachyon inflation in an AdS braneworld

- Dimensionless:

$$h = H / k,\ \phi = \Phi / (k\sqrt{\lambda}),$$

$$\pi_\phi = \Pi_\Phi / (k^2\sqrt{\lambda})),\ \theta = k\Theta,\quad \pi_\theta = \Pi_\Theta / (k^4\lambda)$$

$$\dot{\phi} = \pi_\phi$$

$$\dot{\theta} = \frac{\theta^4 \psi \pi_\theta}{\sqrt{1+\theta^8\pi_\theta^2/\psi}}$$

$$\dot{\pi}_\phi = -3h\pi_\phi - \frac{\psi}{2\theta^2}\frac{4+3\theta^8\pi_\theta^2/\psi}{\sqrt{1+\theta^8\pi_\theta^2/\psi}}\eta'$$

$$\dot{\pi}_\theta = -3h\pi_\theta + \frac{\psi}{\theta^5}\frac{4-3\theta^{10}\eta\pi_\theta^2/\psi}{\sqrt{1+\theta^8\pi_\theta^2/\psi}}$$

$$\kappa^2 = 8\pi\lambda G k^2$$

$$\dot{h} = -\frac{\kappa^2}{2}(\bar{\rho}+\bar{p})\left(1+\frac{\kappa^2}{6}\bar{\rho}\right)$$

$$\dot{N} = h$$

Tachyon inflation in an AdS braneworld

- Slow-roll parameters:

$$\epsilon_1 \simeq \frac{8\theta^2}{\kappa^2}\left(1+\frac{\kappa^2}{6\theta^4}\right)\left(1+\frac{\kappa^2}{12\theta^4}\right)^{-2}$$

$$\epsilon_2 \simeq \frac{8\theta^2}{\kappa^2}\left(1+\frac{\kappa^2}{12\theta^4}\right)^{-2}\left[1+\frac{\kappa^2}{4\theta^4}-\frac{\kappa^2}{6\theta^4}\left(1+\frac{\kappa^2}{6\theta^4}\right)^{-1}\right]$$

- Observational parameters:

$$r = 16\epsilon_1(\theta_i)\left[1-\frac{1}{6}\epsilon_1(\theta_i)+C\epsilon_2(\theta_i)\right]$$

$$n_s = 1-2\epsilon_1(\theta_i)-\epsilon_2(\theta_i)-\left[2\epsilon_1^2(\theta_i)+\left(2C+\frac{8}{3}\right)\epsilon_1(\theta_i)\epsilon_2(\theta_i)+C\epsilon_2(\theta_i)\epsilon_3(\theta_i)\right]$$

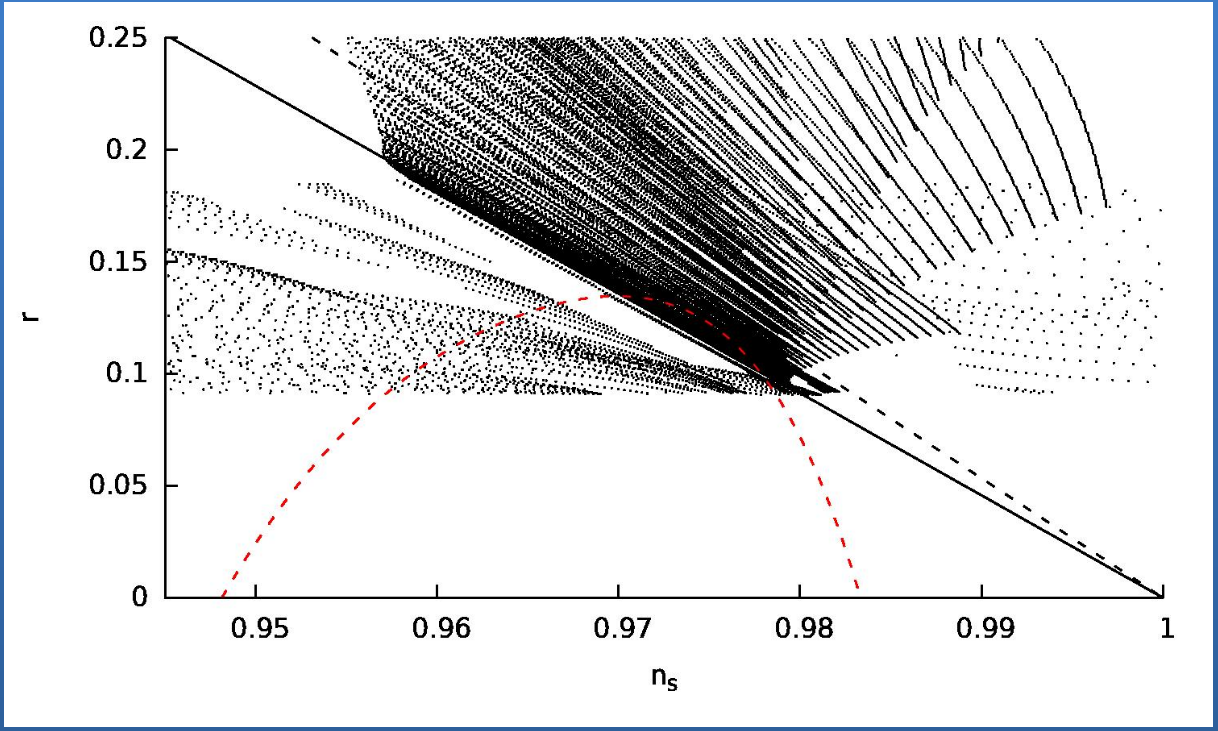
Tachyon inflation in an AdS braneworld

- Some numerical results:

$$60 \leq N \leq 120$$

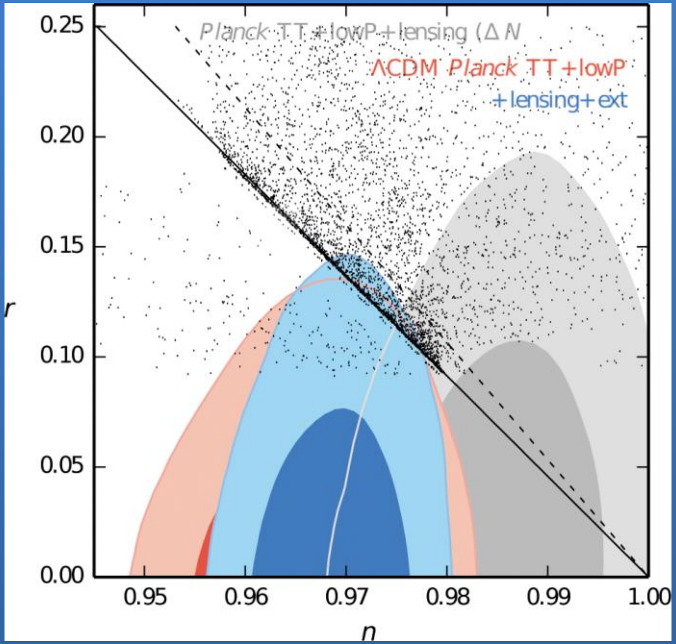
$$1 \leq \kappa \leq 12$$

$$0.05 \leq \phi_0 \leq 0.5$$



Tachyon inflation in an AdS braneworld

- Some numerical results:



$$60 \leq N \leq 120,\ 1 \leq \kappa \leq 12 \text{ and } 0 \leq \phi_0 \leq 0.5$$

Conclusion

- We have investigated a model of inflation based on the dynamics of a D3-brane in the AdS5 bulk of the RSII model. The bulk metric is extended to include the back reaction of the radion excitations.
- The n_s/r relation here is substantially different from the standard one and is closer to the best observational value.
- The model is based on the brane dynamics which results in a definite potential with one free parameter only.
- We have analized the simplest tachyon model. In principle, the same mechanism could lead to a more general tachyon potential if the AdS5 background metric is deformed by the presence of matter in the bulk.

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- This work is supported by the SEENET-MTP Network under the ICTP grant NT-03.
- The financial support of the Serbian Ministry for Education and Science, Projects OI 174020 and OI 176021 is also kindly acknowledged.

THANK YOU!

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