Supersymmetric Bag Model to Unite Gravity with Particle Physics

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Based on: arXiv:1701.01025 and

A.B., *Gravitating Lepton Bag Model* , JETP, v.148 (8), 228 (2015),
A.B., *Stability of the Lepton Bag Model ...*, JETP, v.148(11), 937 (2015), arXiv:1706.02979,
***A.B., Source of the Kerr-Newman solution ...* Phys.Lett. B754, 99 (2016), arXiv:1602.04215.**

"... a realistic model of elementary particles still appears to be a distant dream ... "
John Schwarz, arXiv:1201.0981.

Quantum and Gravity cannot be combined in a unified theory. Gravity requires *field model of particles* for the right side of Einstein equations, $G_{\mu\nu} = 8\pi T_{\mu\nu}$.
Kaluza-Klein model: $^5G_{MN} = 0$ **gives** $^4G_{\mu\nu} = 8\pi T_{\mu\nu}$, **potential** A^μ **and scalar** Φ.
Invisible extra dimensions (KK-modes) are compactified at *Planck scale.*
Superstring theory inherits the KK idea of compactification at Planck scale:
a) 'natural' units, b) invisible extra dimensions, c) weakness of gravity.
Spin deforms space along with mass by frame dragging, or Lense-Thirring effect! Weakness of Gravity is an illusion caused by underestimation of the role of SPIN.

GRAVITY IS NOT WEAK because SPIN of particles enormously exceeds their mass: $J/m = 10^{20} - 10^{22}$ **in dimensionless units** $G = c = \hbar = 1$**.**
Nobody says that gravity is weak in COSMIC because of the great cosmic masses. Similar, gravity is not weak in particle physics because of the *the giant spin/mass ratio* for spinning particles!

Spin shifts gravitational interaction from Planck to Compton scale, so that Gravity and Quantum theory become on the equal footings!

It is confirmed by analysis of the Kerr-Newman solution.
For electron the spin/mass ratio is $\sim 10^{22}$, and spinning Kerr-Newman solution with parameters of an electron deforms space at the Compton distance.

Schwarzschild's estimation of gravitational coupling constant $r_g \sim 2m$.
Kerr geometry indicates strong influence of the SPIN at the Compton distance

$$r_c \sim \hbar/2m_e,$$

contrary to the usually accepted Planck length.
Horizons of Kerr black hole (BH) disappear, displaying naked Kerr singularity which deforms space topologically at the Compton distance.

Singularity is signal of New Physics! Quantum theory is inapplicable on such space. Conflict between Gravity and Quantum theory starts at the Compton scale!
New concept: there is no priority of Quantum theory to Gravity – Einstein-Maxwell Gravity and Quantum theory interact on an equal footing!

No needs to modify Einstein-Maxwell gravity, and the problem of consistency with quantum physics is solved by Supersymmetric bag model – a nonperturbative solution to SUPERSYMMETRIC HIGGS model, which is equivalent to Landau-Ginzburg (LG) field model.

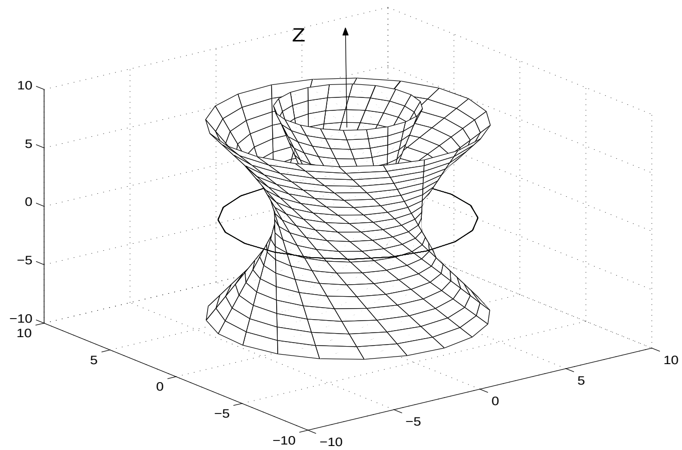
The Kerr-Schild form of metric

$$g_{\mu\nu} = \eta^{\mu\nu} + 2Hk^{\mu}k^{\nu}, \tag{1}$$

vector potential

$$A^{\mu} = ek^{\mu}/(r + ia\cos\theta), \tag{2}$$

and Kerr Theorem, determines Principal Null Congruence k^{μ} in terms of twistors.



For parameters of an electron, horizons of the KN metric disappear, and there appears naked singular ring of Compton radius. GRAVITY gives to electron an EXTENDED VORTEX structure!

Kerr singular ring as a closed string (AB, 1974, D.Ivanenko&AB, 1975).

The light-like closed string looks as a point due to Lorentz contraction! Punsly,1985.

FRAME-DRAGGING along directions of Kerr congruence k^μ. Formation of Wilson loop. Lense-Thyrring (LT) effect of rotation. KN soln.—¿ LT soln. at large distances (Kerr, 1963).

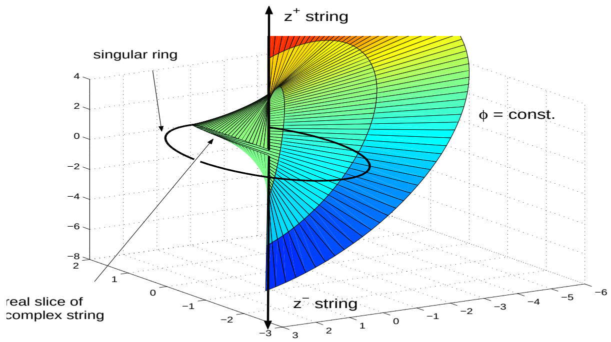


Figure 1: The Kerr congruence and vector potential are dragged by Kerr singular ring, forming a closed Wilson loop.

Lense-Thirring effect – gravitational analog of the Aharonow-Bohm topological effect created by Wilson lines. Loop of the vector potential and traveling waves along the Kerr ring are analogs of KK-modes in string models! Compactification without extra dimensions!

Old problem of the physical source of Kerr-Newman solution. (Bubble and solitonic sols.) KN sol. as model of electron (H.Keres 1967, B.Carter 1968, W.Israel 1970, AB 1974, Ivanenko& AB 1975, A.Krasinski 1978, C.López 1985, I.Dymnikova 2006, AB 2010 etc.)

Gyromagnetic ratio of KN solution, $g = 2$, corresponds to electromagnetic and gravitational field of the Dirac electron (Carter, 1968).

Super-Bag model, AB 2015-2017.

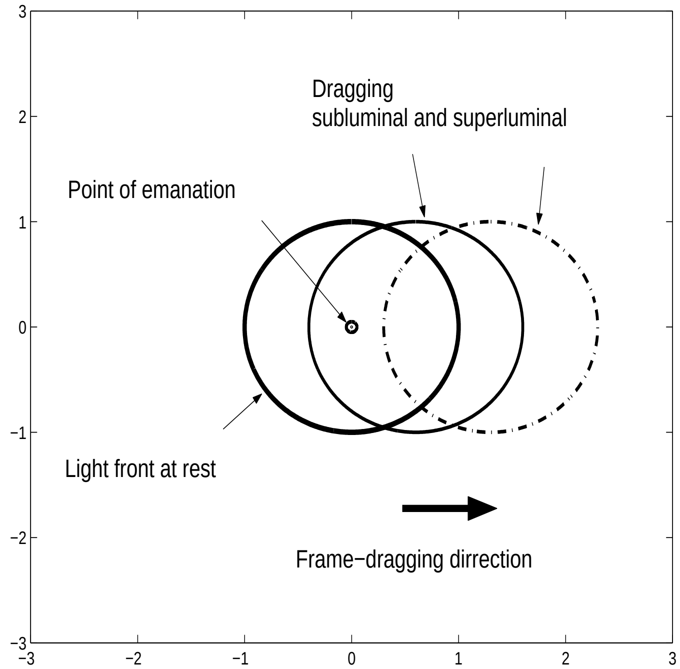


Figure 2: **Frame-dragging as local deformation of light cones.**

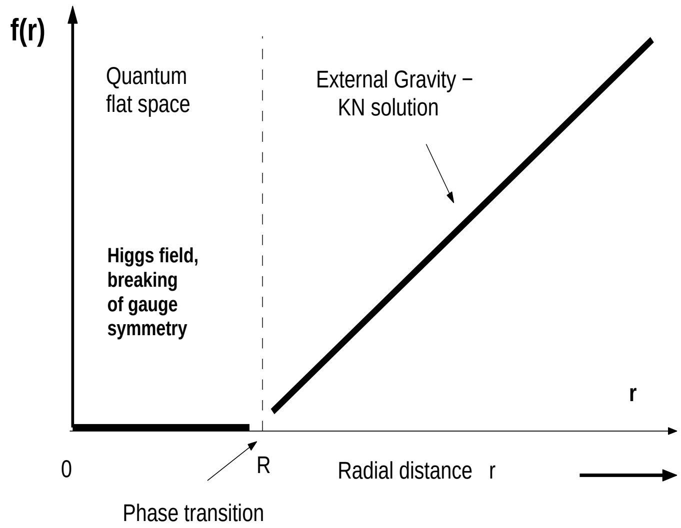


Figure 3: **Bag is built of deformed KN solution suggested by Gürses and Gürsay (JMP, 1975).**

The deformed KN solution $g_{\mu\nu} = \eta_{\mu\nu} + \frac{2f(r)}{r^2+a^2\cos^2\theta} k_\mu k_\nu$ **creates two zones:** **FLAT QUANTUM CORE and external zone of SPINNING KN GRAVITY, which are separated by thin zone of phase transition.**
Position of the boundary R is determined by matching KN metric with flat space

$$g^{(KN)}_{\mu\nu} = \eta_{\mu\nu} + 2H_{(KN)} k_\mu k_\nu, \quad \textbf{where} \quad H_{(KN)} = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta}.$$

At $r = R = e^2/2m$, **we have** $H_{(KN)} = 0$, **and** $g^{(KN)}_{\mu\nu} = \eta_{\mu\nu}$. **Since** r **is spheroidal coordinate, bag takes ellipsoidal form of thickness** R **and radius** a**.**

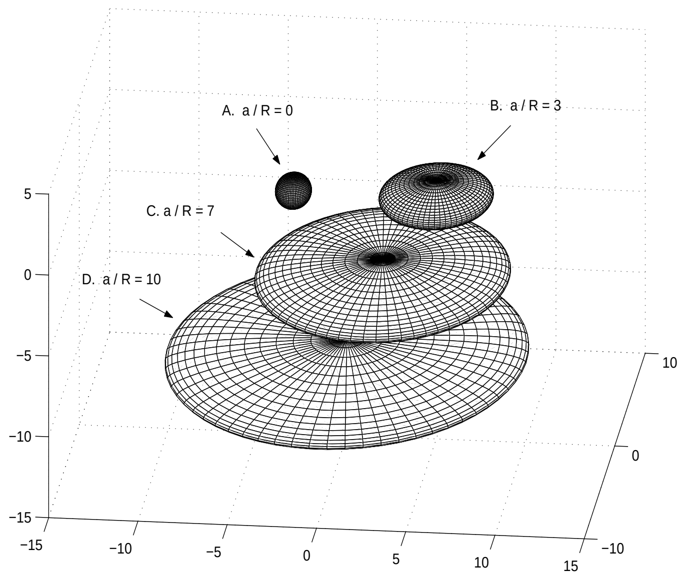


Figure 4: **KN gravity defines shape of bag depending on spin/mass ratio $a = J/m$.**

Boundary of bag is formed by Domain Wall solution to Landau-Ginzburg field model.
LG model is equivalent to Higgs model, and is used in Nielsen-Olesen (NO) dual string model, soliton models and in the MIT and SLAC bag models.
***However*, the usual quartic potential $V = g(H\bar{H} - \sigma^2)^2$ is inappropriate as it creates superconductivity in outer space. In particular, string of NO model**

is a vortex in superconductor, and bag is a ”cavity in superconductor”. To get a model with superconducting core and unbroken outer space, one should use the supersymmetric LG model (AB JETP 2015, 2016; Phys.Lett.B 2016).

SUPERSYMMETRIC scheme of phase transition.

Triplet of the chiral fields $\Phi^{(i)} = \{H, Z, \Sigma\}$, where H is the Higgs field.

Lagrangian $\mathcal{L} = -\frac{1}{4}\sum_{i=1}^{3} F^{(i)}_{\mu\nu} F^{(i)\mu\nu} - \frac{1}{2}\sum_{i=1}^{3} (\mathcal{D}^{(i)}_{\mu}\Phi^{(i)})(\mathcal{D}^{(i)\mu}\Phi^{(i)})^{*} - V$, covariant derivatives $\mathcal{D}^{(i)}_{\mu} = \nabla_{\mu} + ieA^{(i)}_{\mu}$.

Superpotential (suggested by J. Morris, 1996)

$$W = \Phi^{(2)}(\Phi^{(3)}\bar{\Phi}^{(3)} - \eta^2) + (\Phi^{(2)} + \mu)\Phi^{(1)}\bar{\Phi}^{(1)}, \tag{3}$$

determines the potential

$$V(r) = \sum_i |\partial_i W|^2, \tag{4}$$

where $\mathcal{H} \equiv \Phi^{(1)}$ is taken as Higgs field.

Vacuum states $V_{(vac)} = 0$ are determined by the conditions $\partial_i W = 0$. The model yields two vacuum states:

(I) the supersymmetric false-vacuum state inside: $|H| = \eta;\ Z = -\mu;\ \Sigma = 0,$

(II) the vacuum state outside: $|H| = 0;\ Z = 0;\ \Sigma = \eta.$

Higgs field H forms inside the bag the supersymmetric and superconducting vacuum state.

Einstein-Maxwell eqs. are trivially satisfied inside and outside the bag.

The Landau-Ginzburg (LG) field model describes NO model of vortex line in superconductor and is fully equivalent to Higgs mechanism of symmetry breaking. Setting $\Phi^{(1)} \equiv H$ we have

$$\mathcal{L}_{NO} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\mathcal{D}_\mu \Phi)(\mathcal{D}^\mu H)^* - V(|H|).$$

Corresponding eqs. describe concentration of the Higgs field $H(x) = |H| e^{i\chi(x)}$ in the core of particle and its interaction with vector potential:

$$\mathcal{D}^{(1)}_\nu \mathcal{D}^{(1)\nu} H = \partial_{H^*} V, \tag{5}$$

$$\nabla_\nu \nabla^\nu A_\mu = I_\mu = \frac{1}{2} e |H|^2 (\chi,_\mu + e A_\mu). \tag{6}$$

At the rim of disk, $r = e^2/2m, \cos\theta = 0,$ **KN potential is** $A_\mu dx^\mu = A^{max}_\mu dx^\mu = -\frac{2m}{e}(dr - dt - a d\phi).$
Inside superconducting core $I_\mu = 0$, **and from** $\chi,_\mu + e A_\mu = 0$ **and** $e A_t = 2m, \ e A_\varphi = 2ma$, **we obtain** $\chi = -2mt - 2ma\varphi$, **which leads to important consequences:**

(i) closed flux of the vector potential $\oint e A_\varphi d\varphi = -4\pi m a$ **forms a *quantum Wilson loop* leading to quantized angular momentum,** $J = ma = n\hbar/2, \ n = 1, 2, 3, ...$

(ii) phase of the Higgs χ oscillates with frequency $\omega = 2m$ similar to solitonic models of oscillons and Q-balls (G.Rosen 1968, Coleman 1985).

Wilson line of the vector potential is parametrized by periodic phase of the Higgs field creating cylindricity of the model!
KK cylindricity at the Compton scale allows us to do compactification without extra dimensions!

NONPERTURBATIVE BPS-saturated SOLUTION . Large **AB, JETP, v.148, 937(2015), arXiv:1706.02979, AB, Phys.Lett. B754, 99(2016), arXiv:1602.04215.**

Supersymmety and Bogomolnyi bound. Hamiltonian:

$$H^{(ch)} = T_0^{\ 0(ch)} = \frac{1}{2}\sum_{i=1}^{3} \, [\sum_{\mu=0}^{3} |\mathcal{D}_\mu^{(i)}\Phi^i|^2 + |\partial_i W|^2].$$

Kerr's coordinate system $x + iy = (r + ia)e^{i\phi}\sin\theta, \quad z = r\cos\theta, \quad t = \rho - r.$ **Vector potential**

$$A_\mu dx^\mu = -Re \, [(\frac{e}{r + ia\cos\theta})](dr - dt - a\sin^2\theta d\phi). \tag{7}$$

Terms $A_\phi d\phi$ **and** $A_t dt$ **drop out of the Hamiltonian due the constraints**

$$\mathcal{D}_t^{(1)}\Phi^1 = 0, \quad \mathcal{D}_\phi^{(1)}\Phi^1 = 0, \tag{8}$$

consistent with (i) and (ii). The rest is reduced to integral over variable $r.$

$$H^{(ch)} = T_0^{\ 0(ch)} = \frac{1}{2}\sum_{i=1}^{3} \, [|\mathcal{D}_r^{(i)}\Phi^i|^2 + |\partial_i W|^2], \tag{9}$$

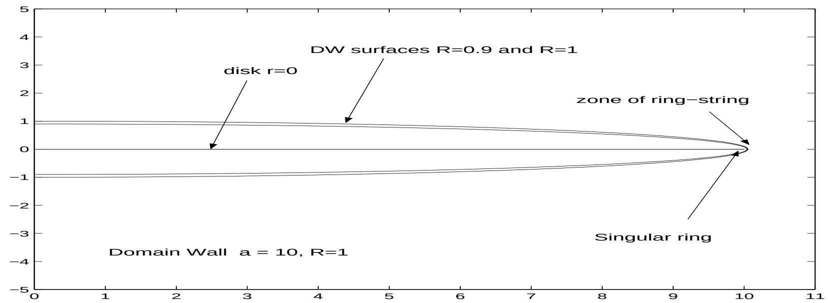


Figure 5: **Axial section of the spheroidal domain wall phase transition**.

Then we use the TRICK suggested by Cvetiĉ & Rey for planar Dom Wall, which WORKS! and allows to transform Hamiltonian to Bogomolnyi form

$$H^{(ch)} = T_0{}^{0(ch)} = \frac{1}{2}\sum_{i=1}^{3} \; [|\mathcal{D}_r^{(i)}\Phi^i - e^{i\chi_i}\partial_i\bar{W}|^2 + 2Re \; e^{-i\chi_i}\partial_i\bar{W}\mathcal{D}_r^{(i)}\Phi^i] \tag{10}$$

The angles χ_i are determined by phase of the oscillating Higgs field

$$\Phi(x) \equiv \Phi^1(x) = |\Phi^1(r)|e^{i\chi(t,\phi)}. \tag{11}$$

It yields $\chi_1 = 2\chi(t,\phi), \;\; \chi_2 = \chi_3 = 0$, and We obtain the Bogomolnyi equations

$$\mathcal{D}_r^{(i)}\Phi^i = \partial W/\partial\Phi^i, \quad \mathcal{D}_r^{(i)}\bar{\Phi}^i = \partial\bar{W}/\partial\bar{\Phi}^i. \tag{12}$$

Hamiltonian turns into full differential ($\mathcal{D}_r \to \partial_r$ due structure of W)

$$H^{(ch-r)} = Re \; (\partial W/\partial\Phi^i)\partial_r\Phi^i = \partial W/\partial r. \tag{13}$$

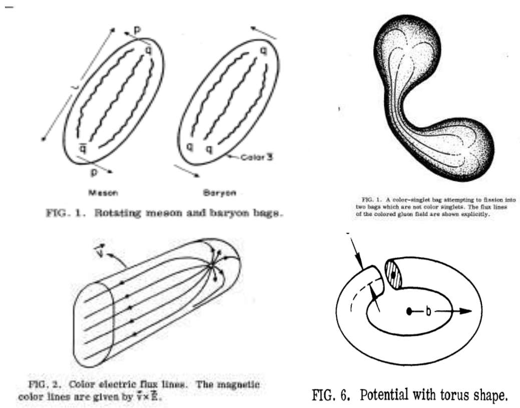
Using the Kerr coordinate system, and $\Delta W = W(R+\delta) - W(R-\delta) = -\mu\eta^2$, **we obtain**

$$\delta M_{bag} = 2\pi\Delta W \int_{-1}^{1} dX(R^2 + a^2X^2) = 4\pi(R^2 + \frac{1}{3}a^2)\Delta W. \tag{14}$$

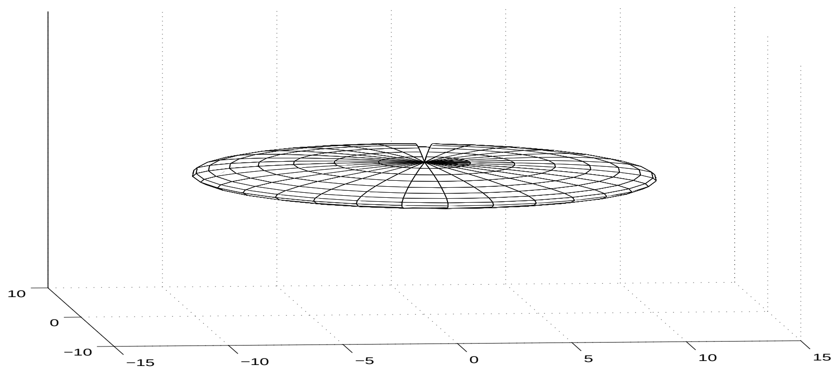
BPS-saturated solution $\Rightarrow$ **Stability.**

STRINGY STRUCTURES

BAGs are soft and elastic. While rotating they take shape of a string. The meson bag turns in fluxtube (K. Johnson and C. B. Thorn, Stringlike solutions of the bag model, PRD 13, 1934 (1976); Chodos et al. PRD 9, 3471 (1974).)



Kerr-Newman bag creates circular string at the border of oblate bag.



Kerr's circular string

The Kerr singular ring as a closed string – "gravitational waveguide" for traveling EM waves (pp-waves), (A.B. 1974. A.B.& Ivanenko 1975.) Similar to fundamental string solutions to low energy string theory (A.Sen NPB, 1992; PRL 1995; Strings as solitons, Dabholkar at al. 1995, AB PRD 1995, AB PRD 2003).

Types of excitations:

I. Vibrations – bosonic excitations.
II. Electromagnetic traveling waves.
III. Winding modes of vector potential – flux-tube tension.
IV. Surface current along border of the disk – spinor excitations.
IV. Complex structure of the Kerr geometry has a complex N=2 superstring embedded in the complex structure of 4D Kerr geometry (A.B. 2014).

Circular string of the Kerr geometry is LIGHTLIKE.

An external observer will see the lightlike string as a the point-like source. Lightlike string forms worldline – not worldsheet. The lightlike closed pp-string shrinks to point by Lorentz contraction! (Punsly 1985, Arcos & Pereira, 2006, A.B. 2009.) To form a worldsheet, the "left" lightlike mode should be completed by "right" mode.

Phase of the oscillating Higgs field φ plays the role of periodic coordinate of compactification, and together with time t of the oscillating Higgs field $H = |H|e^{i2m(t+a\varphi)}$ they form parametrization of the worldsheet.

Excitations:

Stationary Kerr-Newman solution $\psi = e$ creates a frozen electromagnetic wave along boundary of the bag defined by $H(r, \psi) = 0$. Electromagnetic excitations create traveling waves.

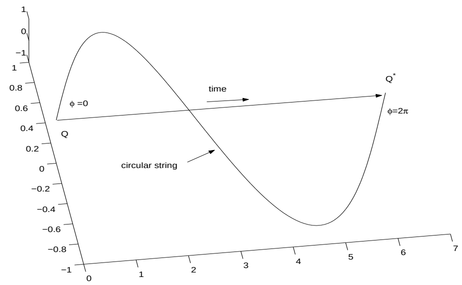
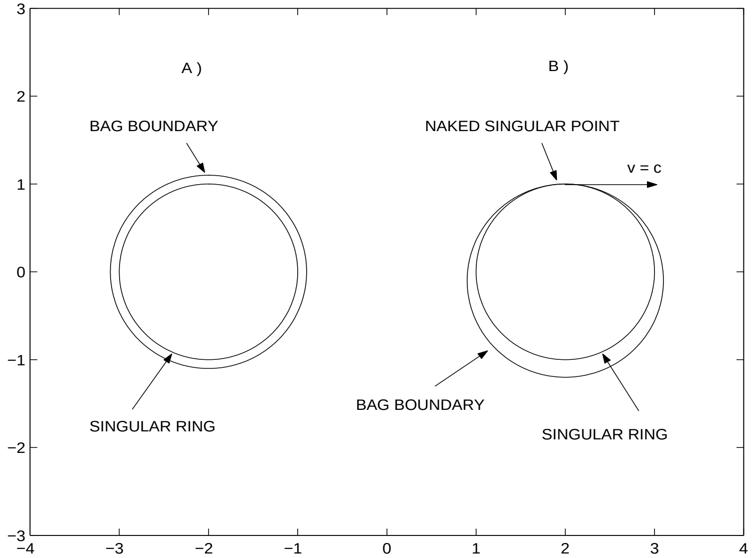


Figure 6: **The circular left mode, formed by traveling wave along the KN string, is to be completed by the time-like right mode, formed by the frozen traveling wave of the stationary KH solution.**

Boundary of bag is determined by "zero gravity surface" $H = 0$, where

$$H = \frac{mr - |\psi|^2/2}{r^2 + a^2 \cos^2 \theta} \ . \tag{15}$$

Condition $H = 0$ determines boundary of disk $R = |\psi|^2/2m$, which acts as cut-off for EM field.



The lowest exact solution

$$\psi = e(1 + \frac{1}{Y} e^{i\omega\tau}) \tag{16}$$

takes in equatorial plane $\cos\theta = 0$ **the form** $\psi = e(1 + e^{-i(\phi - \omega t)})$, **and the cut-off parameter**

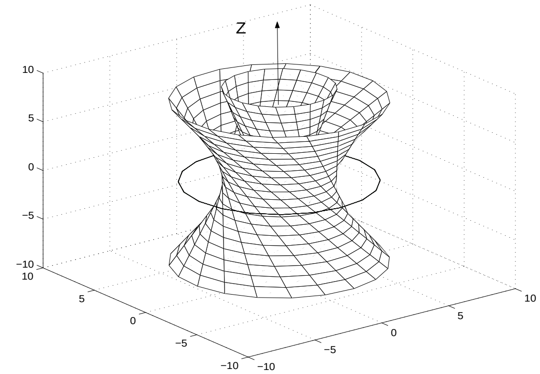
$$R = |\psi|^2/2m = \frac{e^2}{m}(1 + \cos(\phi - \omega t))$$

depends on $\phi - \omega t$.

Vanishing R **at** $\phi = \omega t$ **creates singular pole which circulates along the ring-string. Closed string turns into an open string with singular end points.**

There appears a circulating quark-antiquark pair. Bag-string-quark system – analogue to D2-D1-D0-brane of string-M-theory.

Consistent embedding of the Dirac equation in twistorial structure of the Kerr-Schild geometry.



Algebraically special KN solution – all fields are collinear to Principal Null Directions k^μ of the Kerr congruence k^μ.

Metric of the Kerr-Newman solution: $g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu$ **and vector potential** $A^\mu_{KN} = Re\frac{e}{r+ia\cos\theta}k^\mu$. **The null directions k^μ determine a collinear spinor field.**

THE KERR THEOREM: Kerr congruence has two solutions $k^\pm_\mu$ creating two metrics $g^\pm_{\mu\nu} = \eta_{\mu\nu} + 2Hk^\pm_\mu k^\pm_\nu$. TWOSHEETED Kerr space!

Geodesic and Shear-free congruences are obtained as analytic solutions of the equation $F(T^a) = 0$, where F is a holomorphic function of the projective twistor coordinates in CP^3, $T^a = \{Y, \quad \zeta - Yv, \quad u + Y\bar\zeta\}$.

Projective coordinate $Y = \phi_1/\phi_0$, is equivalent to Weyl spinor ϕ_α.

TWISTOR $\Leftrightarrow$ SPINOR relation is origin of the consistent Dirac field.

DIRAC EQUATION splits in the Weyl representation into two equations

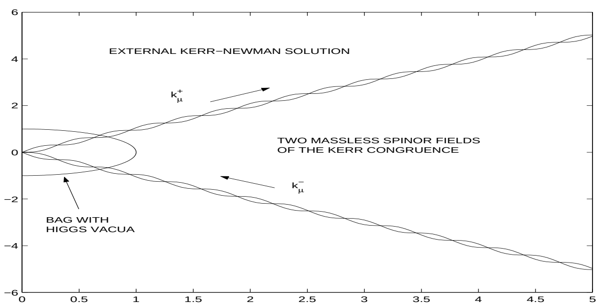
$$\sigma^{\mu}_{\alpha\dot{\alpha}} i\partial_\mu \bar{\chi}^{\dot{\alpha}} = m\phi_\alpha, \quad \bar{\sigma}^{\mu\dot{\alpha}\alpha} i\partial_\mu \phi_\alpha = m\bar{\chi}^{\dot{\alpha}}, \tag{17}$$

the "left-handed" and "right-handed" electron fields, Weyl spinors.

Two antipodally conjugate solutions of the Kerr theorem $Y^+ = -1/\bar{Y}^-$ determine two Weyl spinors ϕ^α and $\bar{\chi}_{\dot{\alpha}}$, corresponding to $Y^+ = \phi_1/\phi_0$ and $Y^- = \bar{\chi}^{\dot{1}}/\bar{\chi}^{\dot{0}}$,

$$\phi_\alpha = \begin{pmatrix} e^{-i\phi/2}\cos\frac{\theta}{2} \\ e^{i\phi/2}\sin\frac{\theta}{2} \end{pmatrix}, \quad \bar{\chi}^{\dot{\alpha}} = \begin{pmatrix} -e^{-i\phi/2}\sin\frac{\theta}{2} \\ e^{i\phi/2}\cos\frac{\theta}{2} \end{pmatrix}, \tag{18}$$

which are aligned to different $k^{\mu\pm}(x)$ and different metrics $g^\pm_{\mu\nu} = \eta_{\mu\nu} + 2H_{(KN)} k^\pm_\mu k^\pm_\nu$.
The "left" and "right" spinors should be placed on different sheets of metric.



Inside the bag the Weyl spinors are united into Dirac bispinor Ψ, and Dirac equation $(\gamma^\mu\partial_\mu + m)\Psi(x) = 0$, acquires mass $m(x) \equiv g\mathcal{H}(x)$ from the Higgs condensate $\mathcal{H}(x)$.

SUPER-BAG – nonperturbative analog to Wess-Zumino SuperQED model. Super-QED forms a bridge to perturbative QED of the electron!
Supersymmetric perturbation theory is developed as a direct extension of the ordinary perturbation theory.
Φ^i **become chiral fields in the component form** $\Phi_i(y) = A_i(y^\mu) + \sqrt{2}\theta\psi_i(y^\mu) + \theta\theta F_i(y^\mu)$.
Kinetic term super-QED has two chiral fields Φ_+ **and** Φ_-,

$$\mathcal{L}_{kinQED} = \frac{1}{4} Re \int d^4x\, d^2\theta\, W^a W_a + \int d^4x\, d^4\theta\, (\Phi_+^+ e^{eV} \Phi_+ + \Phi_-^+ e^{-eV} \Phi_-), \tag{19}$$

and potential term is formed as the sum of the chiral and anti-chiral parts $W + W^+$.
The Feynman rules are stated in terms of superfield vertices and propagators with miraculous cancellations between component diagrams. (Wess and Bagger "Supersymmetry and Supergravity".)

GENERALIZATION: Nonperturbative Super-QED field model is constructed as unification of the kinetic part of super-QED with potential of the bosonic super-Bag. In notations $\Phi_+ = \Phi$, $\Phi_- = \bar{\Phi}$, and $\Phi_1 = \Sigma$, $\Phi_2 = \bar{\Sigma}$, and $\Phi_0 = Z$, superpotential takes the form $W(\Phi_i) = \Phi_0(\Phi_1\Phi_2 - \eta^2) + (\Phi_0 + \mu)\Phi_+\Phi_-$.
Nonperturbative Super-QED bag model of dressed electron is matched with QED and principles of the SM. It shows that the Compton zone of the consistent with gravity dressed electron must have the form a superconducting disk, built from supersymmetric vacuum state of the Higgs field. It contains the light-like string on perimeter of the bag and circulating pole. The known zitterbewegung of the Dirac electron acquires natural explanation as consequence of the traveling wave solutions.

CONCLUSION:

- **Spin is gravitating, and great spin/mass ratio of particles shifts-up the scale of gravitational interaction from Planck to Compton lengths, leading to a new concept, in which Quantum theory and Gravity appear on an equal footing.**

- **The supersymmetric Landau-Ginzburg field model resolves conflict between quantum theory, forming a bag where quantum theory is separated from gravity by domain wall boundary.**

- **The Super-bag model based on Kerr-Newman solution has many features of the bag models for hadrons, in particular:**

  a) bag is soft and deformable, creating a circular string at the bag border;

  b) end-points of the string are coupled to an analog of lightlike quark;

  c) Kerr congruence creates Weyl spinors of the Dirac equation which acquires a mass term from the Yukawa coupling.

- **The supersymmetric Higgs (or LG) model model is equivalent to the Wess-Zumino Super-QED model, indicating a link with quantum electrodynamics, AB, arXiv:1701.01025.**

THANK YOU FOR YOUR ATTENTION!

X. Ji, Gauge-Invariant Decomposition of Nucleon Spin, PRL 78,610 (1997),

$$J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{0jk}, \quad M^{\alpha\mu\nu} = T^{\alpha\nu}x^\mu - T^{\alpha\mu}x^\nu,$$

$$T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}.$$

$$T_q^{\mu\nu} = \frac{1}{2}[\bar{\Psi}\gamma^{(\mu}i\overrightarrow{\mathcal{D}}^{\nu)}\Psi + \bar{\Psi}\gamma^{(\mu}i\overleftarrow{\mathcal{D}}^{\nu)}\Psi], \qquad T_g^{\mu\nu} = \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F_\alpha^\nu.$$

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M. Burkardt, The Nucleon Spin Sum Rule, arXiv:1304.0281

The Ji decomposition, frame-independent and manifestly gauge-invariant $\frac{1}{2} = \frac{1}{2}\sum_q(q^\dagger\Sigma^z q + L_q^z) + J_g^z$, where $L_q^z \sim q^\dagger(\vec{r}\times i\vec{\mathcal{D}})^z q$ the expectation value of a manifestly gauge invariant local operator $iD = i\partial - gA$. It has received considerable attention for its relation to generalized parton distributions (GPDs) and experimental probes, it is not natural in the language of parton physics. (Ji-Zhang PLB 2015).

Jaffe and Manohar have proposed an alternative decomposition of the nucleon spin, motivated from a free-field expression of QCD angular momentum boosted to the infinite momentum frame (IMF), defined in the light-cone gauge $A^+ = 0$. It has does have a partonic interpretation $\frac{1}{2} = \frac{1}{2}\sum_q(\Delta_q + \mathcal{L}_q^z) + \Delta G + \mathcal{L}_g^z$, where the first term $\Delta_q = q_+^\dagger\gamma^5 q_+$ and third term ΔG are the 'intrinsic' contributions (no factor of $\sim r\times$), and have a physical interpretation as quark and gluon spin respectively, while the second term $\mathcal{L}_q^z = q_+^\dagger(\vec{r} \times i\vec{\partial})^z q_+$ and fourth term $\mathcal{L}_g^z$ can be identified with the quark/gluon OAM. Conceptual problem: all terms except the first one are gauge dependent, and it is unclear why the light-cone gauge operator is measurable in physical experiments. ===

The stress-energy tensor of bag model may be decomposed into pure em part and contributions from the chiral fields

$$T_{\mu\nu}^{(tot)} = T_{\mu\nu}^{(em)} + \delta_{i\bar{j}}(\mathcal{D}_\mu^{(i)}\Phi^i)\overline{(\mathcal{D}_\nu^{(j)}\Phi^j)} - \frac{1}{2}g_{\mu\nu}[\delta_{i\bar{j}}(\mathcal{D}_\lambda^{(i)}\Phi^i)\overline{(\mathcal{D}^{(j)\lambda}\Phi^j)} + V]. \tag{20}$$

In the external vacuum state, for $r > r_0 + \xi$, we have $V^{ext} = 0$. The unique nonzero chiral field Σ is constant, and therefore, all the derivatives $\mathcal{D}_\mu^{(i)}\Phi^{(i)}$ vanish. As a result $T_{\mu\nu}^{(tot)}$ is reduced to $T_{\mu\nu}^{(em)}$, and we obtain the usual Einstein-Maxwell field equations which for the external KN electromagnetic field correspond to the external KN solution.

For interior of the bubble we have $V^{int} = 0$, and the unique nonconstant Higgs field is $\Phi(x) = |\Phi(x)|e^{i\chi(x)}$. The Lagrangian () is reduced to (**??**) with $V(r) = 0$ and leads to equations

$$\mathcal{D}_\nu\mathcal{D}^\nu\Phi = 0, \quad \nabla_\nu\nabla^\nu A_\mu = I_\mu = \frac{1}{2}e|\Phi|^2(\chi,_\mu + eA_\mu). \tag{21}$$