Aspects of Braneworld Cosmology and Holography

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Basic idea

Braneworld cosmology is based on the scenario in which matter is confined on a brane in a higher dimensional bulk with only gravity allowed to propagate in the bulk. The brane can be placed, e.g., at the boundary of a 5-dim asymptotically Anti de Sitter space (AdS_5)

Why AdS?

Anti de Sitter space is dual to a conformal field theory at its boundary (AdS/CFT correspondnce)

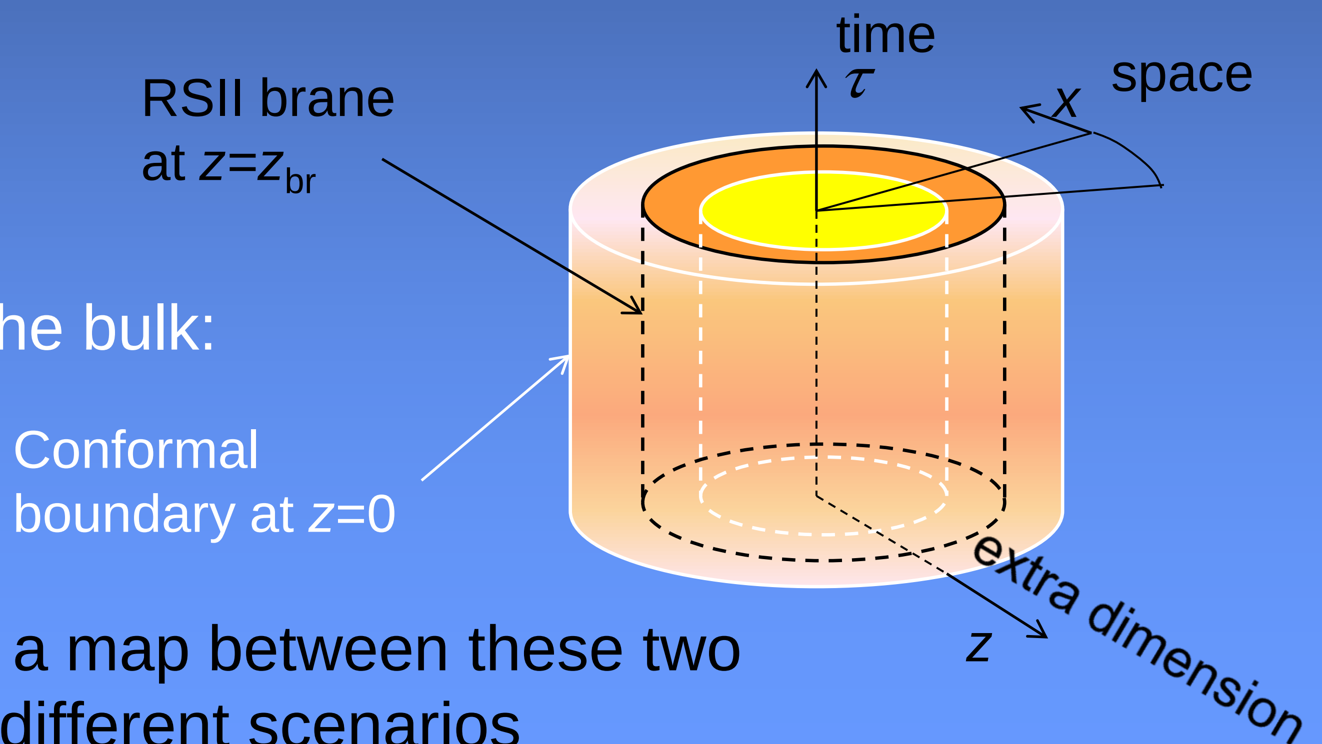
AdS_5 is a maximally symmetric solution to Einstein's equations with negative cosmological constant. In 4+1 dimensions the symmetry group is $AdS_5 \equiv SO(4,2)$

The 3+1 boundary conformal field theory is invariant under conformal transformations: Poincare + dilatations + special conformal transformation = conformal group ≡ SO(4,2)

We will consider two types of braneworlds

1. **Holographic braneworld:** a 3-brane located at the boundary of the asymptotic AdS_5. The cosmology is governed by matter on the brane in addition to the boundary CFT
2. **Randall-Sundrum (RS II) braneworld**: a 3-brane located at a finite distance from the boundary of AdS_5. The RSII model was proposed as an alternative to compactification of extra dimensions

Foliation of the bulk:



There exists a map between these two substantially different scenarios

This talk is based on:

N.B., Phys Rev D 93 (2016) [arXiv:1511.07323]

and related earlier works

E. Kiritsis, JCAP **0510** (2005) [hep-th/0504219].
P.S. Apostolopoulos, G. Siopsis and N. Tetradis,
Phys. Rev. Lett. **102** (2009) [arXiv:0809.3505]
P. Brax and R. Peschanski, Acta Phys. Polon. **B 41** (2010)
[arXiv:1006.3054]

Outline

1. Randall–Sundrum model - basics
2. Connection with AdS/CFT
3. Holographic cosmology
4. Holographic map
5. Effective energy density
6. Conclusins & Outlook

1. Randall-Sundrum model

RS model is a 4+1-dim. universe with AdS_5 geometry containing **two** 3-branes with opposite brane tensions separated in the 5th dimension.

$$S = S_{\text{bulk}} + S_{\text{GH}} + S_{\text{br}1} + S_{\text{br}2}$$

bulk action

$$S_{\text{bulk}} = \frac{1}{8\pi G_5} \int d^5x \sqrt{\det G} \left(-\frac{R^{(5)}}{2} - \Lambda_5 \right)$$

Gibbons-Hawking term

$$S_{\text{GH}} = \frac{1}{8\pi G_5} \int_{\Sigma} d^5x \sqrt{-\det h} K$$

K – trace of the extrinsic curvature tensor

$$K_{ab} = h_a^c h_b^d n_{d;c}$$

brane action

$$S_{\text{br}} = -\sigma \int_{\Sigma} d^4x \sqrt{-h} + \int_{\Sigma} d^4x \sqrt{-h}\, \mathcal{L}_{\text{matt}}$$

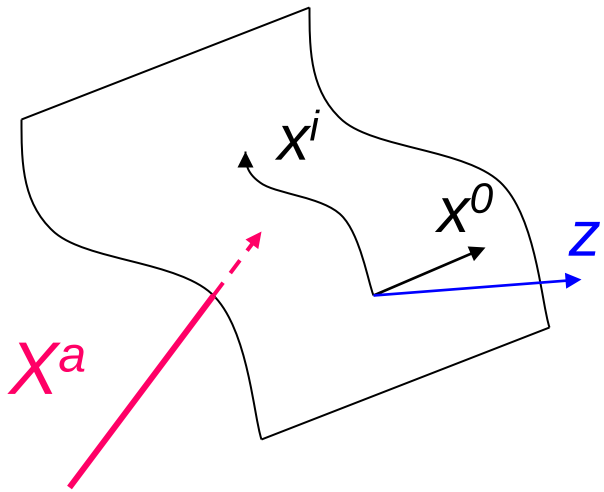
P-brane is a *p*-dim. object that generalizes the concept of membrane (2-brane) or string (1-brane)

Nambu-Goto action for a 3-brane embedded in a 4+1 dim space-time (bulk)

$$S_{\mathrm{br}} = -\sigma \int d^4x \sqrt{-\det(h)}$$

where $h_{\mu\nu}$ is the induced metric

$$h_{\mu\nu} = G_{ab} \frac{\partial X^a}{\partial x^\mu} \frac{\partial X^b}{\partial x^\nu}$$



G_{ab} – metric in the bulk

X^a – coordinates in the bulk

$a,b=0,1,2,3,4$

x^μ – coordinates on the brane

$\mu,\nu=0,1,2,3$

AdS bulk is a space-time with negative cosmological constant:

$$\Lambda^{(5)} = -\frac{6}{\ell^2}$$

ℓ - curvature radius of AdS_5

Various coordinate representations:

Fefferman-Graham coordinates

$$ds^2_{(5)} = G_{\mu\nu}dx^\mu dx^\nu = \frac{\ell^2}{z^2}(g_{\mu\nu}dx^\mu dx^\nu - dz^2)$$

$$z = \ell e^{y/\ell}$$

Gaussian normal coordinates

$$ds^2_{(5)} = e^{-2\ell y} g_{\mu\nu}dx^\mu dx^\nu - dy^2$$

Schwarzschild coordinates (static, spherically symmetric)

$$ds^2_{(5)} = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2 d\Omega^2_\kappa$$

$$f(r) = \frac{r^2}{\ell^2} + \kappa - \mu\frac{\ell^2}{r^2}$$

$$d\Omega^2_\kappa = d\chi^2 + \frac{\sin^2\sqrt{\kappa}\chi}{\kappa}d\Omega^2$$

$$\kappa = \begin{cases} +1 & \text{closed spherical} \\ 0 & \text{open flat} \\ -1 & \text{open hyperbolic} \end{cases}$$

$$\mu = \frac{8G_5 M_{\mathrm{bh}}}{3\pi\ell^2}$$

The coordinates *r* and *z* are related via

$$\frac{r^2}{\ell^2} = \frac{\ell^2}{z^2} - \frac{\kappa}{2} + \frac{\kappa^2 + 4\mu}{16}\frac{z^2}{\ell^2}$$

Second Randall-Sundrum model (RS II)

RS II was proposed as an alternative to compactification of extra dimensions. If extra dimensions were large that would yield unobserved modification of Newton's gravitational law. Experimental bound on the volume of *n* extra dimensions

$$V^{1/n} \leq 0.1\,\mathrm{mm}$$

Long *et al*, Nature **421** (2003).

RSII brane-world does not rely on compactification to localize gravity at the brane, but on the curvature of the bulk ("warped compactification"). The negative cosmological constant $\Lambda^{(5)}$ acts to "squeeze" the gravitational field closer to the brane. One can see this in Gaussian normal coordinates on the flat brane at $y = 0$, for which the AdS_5 metric takes the form

$$ds^2_{(5)} = e^{-2y/\ell}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

warp factor

L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999)

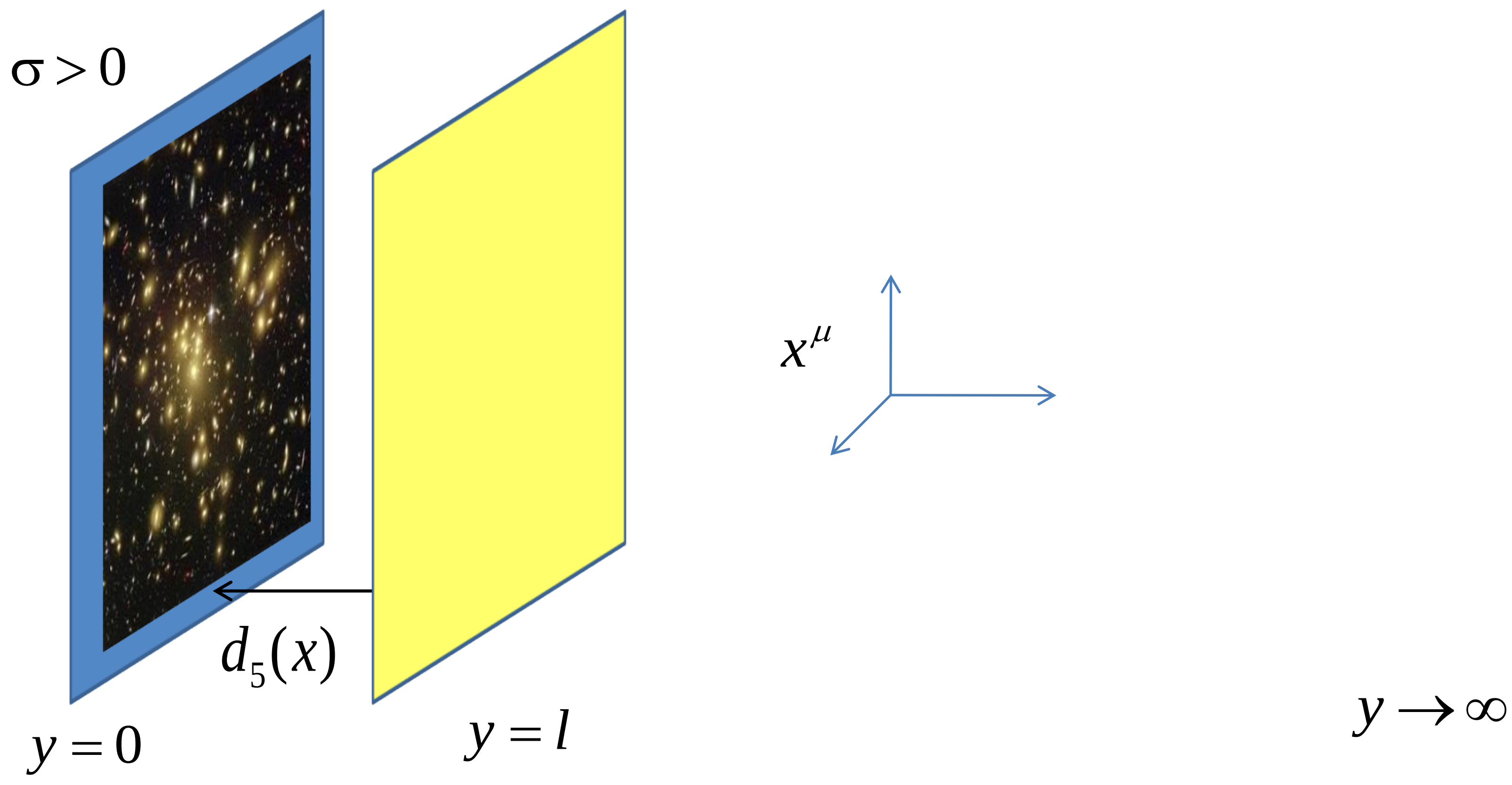
"Sidedness"

In the original RSII model one assumes the Z_2 symmetry

$$z \leftrightarrow z_{\mathrm{br}}/z \qquad \text{or} \qquad y - y_{\mathrm{br}} \leftrightarrow y_{\mathrm{br}} - y$$

so the region $0 < z \le z_{\mathrm{br}}$ is identifed with $z_{\mathrm{br}} \le z < \infty$ so the observer brane is at the fixed point $z = z_{\mathrm{br}}$. The braneworld is sitting between two patches of AdS_5, one on either side, and is therefore dubbed "two-sided". In contrast, in the "one-sided" RSII model the region $0 < z \le z_{\mathrm{br}}$ is simply cut off. 1-sided and 2-sided versions are equivalent from the point of view of an observer at the brane.

In RSII observers reside on the positive tension brane at y=0 and the negative tension brane is pushed off to infinity in the fifth dimension.



The Planck mass scale is determined by the curvature of the five-dimensional space-time

$$\frac{1}{G_{\mathrm{N}}} = \frac{\gamma}{G_5}\int_0^{\infty} e^{-2y/\ell}\,dy = \frac{\gamma\ell}{2G_5} \qquad \gamma = \begin{cases} 1 & \text{one-sided} \\ 2 & \text{two-sided} \end{cases}$$

One usually imposes the RS fine tuning condition

$$\sigma = \sigma_0 \equiv \frac{3\gamma}{8\pi G_5 \ell} = \frac{3}{8\pi G_{\mathrm{N}} \ell^2}$$

which eliminates the 4-dim cosmological constant.

Bound on the AdS_5 curvature radius ℓ:

The classical 3+1 dim gravity is altered on the RSII brane
For $r \gg \ell$ the Newtonian potential of an isolated source on the brane is given by

$$\Phi(r) = \frac{G_{\mathrm{N}} M}{r}\left(1 + \frac{2\ell^2}{3r^2}\right)$$

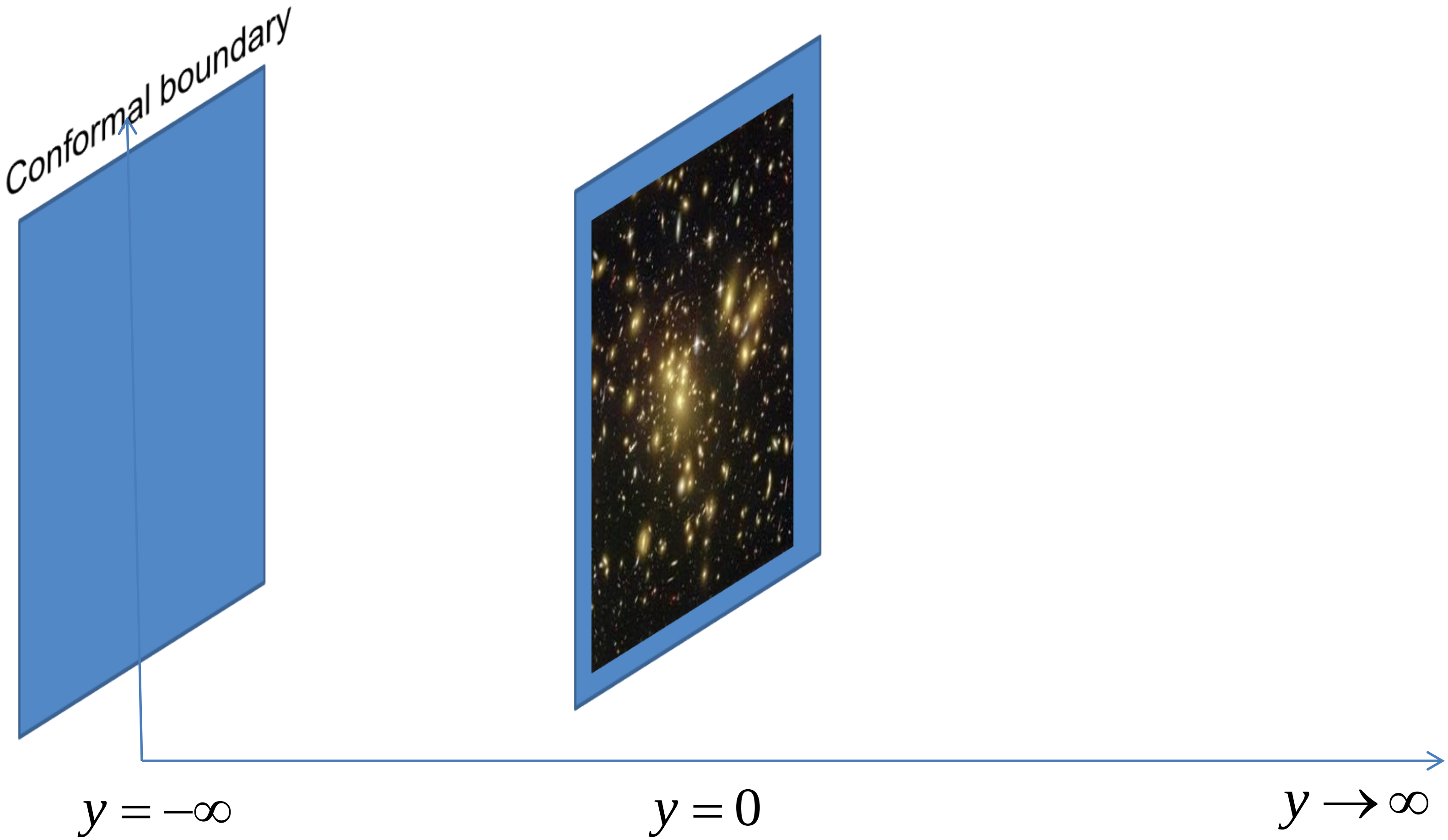
J. Garriga and T. Tanaka, Phys. Rev. Lett. 84, 2778 (2000)

Table top tests of Long et al find no deviation of Newton's potential and place the limit

$$\ell < 0.1\,\mathrm{mm} \quad \text{or} \quad \ell^{-1} > 10^{-12}\,\mathrm{GeV}$$

RSII Cosmology – Dynamical Brane

Cosmology on the brane is obtained by allowing the brane to move in the bulk. Equivalently, the brane is kept fixed at y=0 while making the metric in the bulk time dependent.



Consider a time dependent brane hypersurface defined by

$$r - a(t) = 0$$

in AdS-Schwarzschild background. The induced line element on the brane is

$$ds_{\text{ind}}^2 = n^2(t)dt^2 - a^2(t)d\Omega_k^2$$

where

$$n^2 = f(a) - \frac{(\partial_t a)^2}{f(a)}, \qquad f(a) = \frac{a^2}{\ell^2} + \kappa - \mu\frac{\ell^2}{a^2}$$

$$d\Omega_\kappa^2 = d\chi^2 + \frac{\sin^2\sqrt{\kappa}\chi}{\kappa}\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

The junction conditions on the brane with matter

$$K_{\mu\nu}\big|_{r=a-\epsilon} = \frac{8\pi G_5}{3\gamma}(\sigma g_{\mu\nu} + 3T_{\mu\nu})$$

yield

$$\frac{(\partial_t a)^2}{n^2a^2} + \frac{f}{a^2} = \frac{1}{\ell^2\sigma_0^2}(\sigma+\rho)^2$$

Hubble expansion rate on the brane

The Friedmann equations on the brane are modified.

Imposing a fine tuning of the brane tension $\sigma = \sigma_0 \equiv 3/(8\pi G_N \ell^2)$ one finds

$$\mathcal{H}^2 = \frac{8\pi G_N}{3}\rho + \boxed{\left(\frac{4\pi G_N \ell}{3}\right)^2 \rho^2 + \frac{\mu \ell}{a^4}}$$

where

$$\mathcal{H}^2 = H^2 + \frac{\kappa}{a^2}$$

$$= \frac{(\partial_t a)^2}{a^2 n^2} + \frac{\kappa}{a^2}$$

Quadratic deviation from the standard FRW.
Decays rapidly as $\sim a^{-8}$ in the radiation epoch

dark radiation
due to a black hole in the bulk – should not exceed 10% of the total radiation content in the epoch of BB nucleosynthesis

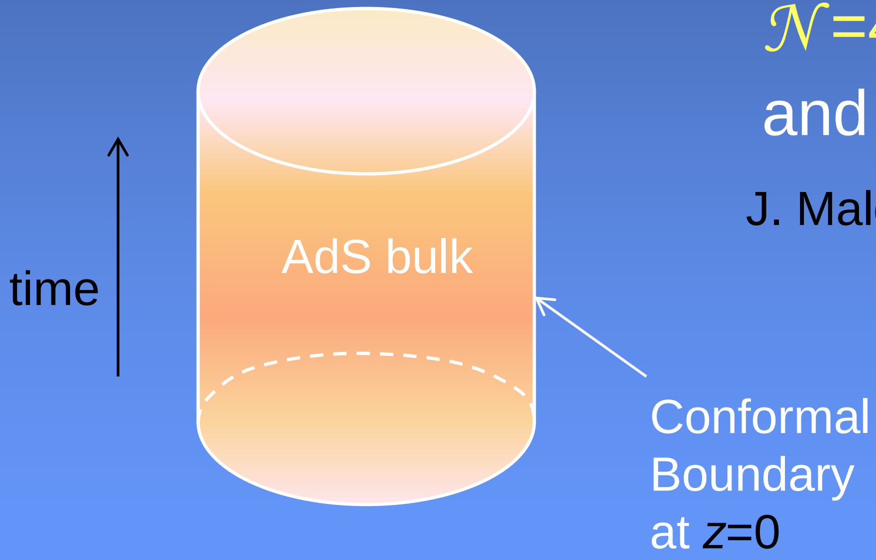
RSII cosmology is thus subject to astrophysical tests

2. Connection with AdS/CFT

AdS/CFT correspondence is a holographic duality between gravity in d+1-dim space-time and quantum CFT on the d-dim boundary. Original formulation stems from string theory:

Equivalence of 3+1-dim
$\mathcal{N}$=4 Supersymmetric YM Theory
and string theory in $AdS_5 \times S_5$

J. Maldacena, Adv. Theor. Math. Phys. **2** (1998)



Examples of CFT:
quantum electrodynamics,
Yang Mills gauge theory,
massless scalar field theory,
massless spin ½ field theory

In the RSII model by introducing the boundary in AdS_5 at $z = z_{br}$ instead of $z = 0$, the model is conjectured to be dual to a cutoff CFT coupled to gravity, with $z = z_{br}$ providing the IR cutoff (corresponding to the UV catoff of the boundary CFT)

In the 1-sided RSII model, by shifting the boundary in the bulk from $z = 0$ to $z = z_{br}$, the model involves a single CFT at the boundary of a single patch of AdS_5.

In the 2-sided RSII model one would instead require two copies of the CFT, one for each of the AdS_5 patches.

M. J. Duff and J. T. Liu, Class. Quant. Grav. 18 (2001); Phys. Rev. Lett. 85, (2000) The on-shell bulk action

$$S_0 = \frac{1}{8\pi G_5}\int d^5x\sqrt{\det G}\left(-\frac{R^{(5)}}{2}-\Lambda_5\right)$$

is IR divergent because physical distances diverge at z=0.
The asymptotically AdS metric near z=0 can be expanded as

$$ds^2_{(5)} = \frac{\ell^2}{z^2}(g_{\mu\nu}dx^\mu dx^\nu - dz^2)$$

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + z^2 g^{(2)}_{\mu\nu} + z^4 g^{(4)}_{\mu\nu} + \cdots$$

Explicit expressions for $g^{(2n)}_{\mu\nu},\ n = 2,4$, in terms of arbitrary $g^{(0)}_{\mu\nu}$ may be found in

de Haro, Solodukhin, Skenderis, Comm. Math. Phys. **217** (2001)

We regularize the action by placing the RSII brane near the AdS boundary, i.e., at $z = \varepsilon\ell$, $\varepsilon<<1$, so that the induced metric is

$$h_{\mu\nu} = \frac{1}{\varepsilon^2}\left(g^{(0)}_{\mu\nu} + \varepsilon^2\ell^2 g^{(2)}_{\mu\nu} + \cdots\right)$$

The bulk splits in two regions: $0 \le z \le \varepsilon\ell$, and $\varepsilon\ell \le z \le \infty$. We can either discard the region $0 \le z \le \varepsilon\ell$ (one-sided regularization, $\gamma = 1$) or invoke the Z_2 symmetry and identify two regions (two-sided regularization, $\gamma = 2$). The regularized bulk action is

$$S^{\mathrm{reg}}_{\mathrm{bulk}} = \gamma S^{\mathrm{reg}}_0 = \frac{\gamma}{8\pi G_5}\int_{z\ge\varepsilon\ell} d^5x\sqrt{\det G}\left(-\frac{R^{(5)}}{2} - \Lambda_5\right) + S_{\mathrm{GH}}$$

The renormalized boundary action is obtained by adding counterterms and taking the limit $\varepsilon \to 0$

$$S_0^{\mathrm{ren}}[g^{(0)}] = \lim_{\varepsilon \to 0} (S_0^{\mathrm{reg}}[G] + S_1[h] + S_2[h] + S_3[h])$$

The necessary counterterms are

$$S_1[h] = -\frac{6}{16\pi G_5 \ell} \int d^4x \sqrt{-h},$$

$$S_2[h] = -\frac{\ell}{16\pi G_5} \int d^4x \sqrt{-h} \left(-\frac{R[h]}{2} \right),$$

$$S_3[h] = -\frac{\ell^3}{16\pi G_5} \int d^4x \sqrt{-h} \frac{\log \epsilon}{4} \left(R^{\mu\nu}[h] R_{\mu\nu}[h] - \frac{1}{3} R^2[h] \right)$$

Hawking, Hertog and Reall, Phys. Rev. D **62** (2000)

Now we demand that the variation with respect to the induced metric $h^{\mu\nu}$ of the total RSII action (the regularized on shell bulk action together with the brane action) vanishes, i.e.,

$$\delta(S_{\rm bulk}^{\rm reg}[h] + S_{\rm br}[h]) = 0$$

Which may be expressed as

matter on the brane

$$\delta\left[\gamma S_0^{\rm ren} - \gamma S_3 - \left(\sigma - \frac{3\gamma}{8\pi\ell G_5}\right)\int d^4x\sqrt{-h} + \int d^4x\sqrt{-h}\,\mathcal{L}_{\rm matt} + \frac{\gamma\ell}{16\pi G_5}\int d^4x\sqrt{-h}\frac{R[h]}{2}\right] = 0.$$

cosmological constant

Einstein Hilbert term

AdS/CFT prescription

$$\delta(S_0^{\rm ren} - S_3) = \frac{1}{2}\int d^4x\sqrt{-h}\langle T_{\mu\nu}^{\rm CFT}\rangle\delta h^{\mu\nu}$$

The variation of the action yields Einstein's equations on the boundary

$$R_{\mu\nu} - \frac{1}{2} R g^{(0)}_{\mu\nu} = 8\pi G_{\mathrm{N}} \left(\gamma \left\langle T^{\mathrm{CFT}}_{\mu\nu} \right\rangle + T^{\mathrm{matt}}_{\mu\nu} \right)$$

where

$$\langle T^{\mathrm{CFT}}_{\mu\nu} \rangle = -\frac{\ell^3}{4\pi G_5} \left\{ g^{(4)}_{\mu\nu} - \frac{1}{8} \left[(\mathrm{Tr} g^{(2)})^2 - \mathrm{Tr}(g^{(2)})^2 \right] g^{(0)}_{\mu\nu} - \frac{1}{2} (g^{(2)})^2_{\mu\nu} + \frac{1}{4} \mathrm{Tr} g^{(2)} g^{(2)}_{\mu\nu} \right\}$$

de Haro et al, Comm. Math. Phys. **217** (2001)

This is an explicit realization of the AdS/CFT correspondence:

the vacuum expectation value of a boundary CFT operator is obtained solely in terms of geometrical quantities of the bulk.

Conformal anomaly

AdS/CFT prescription yields the trace of the boundary stress tensor T^{CFT}

$$\left\langle T^{\mathrm{CFT}\,\mu}{}_{\mu} \right\rangle = \frac{\ell^3}{128\pi G_5}\left(G_{\mathrm{GB}} - C^2\right)$$

$$G_{\mathrm{GB}} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$$ Gauss-Bonnet invariant

$$C^2 = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 2R^{\mu\nu}R_{\mu\nu} + \frac{1}{3}R^2$$ Weyl tensor squared

compared with the general result from field theory

$$\left\langle T^{\mathrm{CFT}\,\mu}{}_{\mu} \right\rangle = bG_{\mathrm{GB}} - cC^2 + b'\Box R$$

The two results agree if we ignore the last term and identify

$$b = c = \frac{\ell^3}{128\pi G_5}$$

Generally $b \neq c$ because

$$b = \frac{n_\mathrm{s} + (11/2)n_\mathrm{f} + 62n_\mathrm{v}}{360(4\pi)^2} \qquad c = \frac{n_\mathrm{s} + 3n_\mathrm{f} + 12n_\mathrm{v}}{120(4\pi)^2}$$

but in $\mathcal{N} = 4$ U(N) super YM theory b=c if

$$n_\mathrm{s} = 6N^2, n_\mathrm{f} = 4N^2, n_\mathrm{v} = N^2$$

The conformal anomaly is correctly reproduced if we identify

$$\frac{\ell^3}{G_5} = \frac{2N^2}{\pi}$$

3. Holographic cosmology

We start from AdS-Schwarzschild static coordinates and make the coordinate transformation $t = t(\tau, z),\ r = r(\tau, z)$.
The line element will take a general form

$$ds_{(5)}^2 = \frac{\ell^2}{z^2}(g_{\mu\nu}dx^\mu dx^\nu - dz^2) = \frac{\ell^2}{z^2}\left[n^2(\tau,z)d\tau^2 - a^2(\tau,z)d\Omega_k^2 - dz^2\right]$$

Imposing the boundary conditions at z=0:

$$n(\tau,0) = 1, \quad a(\tau,0) = a_{\mathrm{h}}(\tau)$$

we obtain the induced metric at the boundary in the FRW form

$$ds_{(0)}^2 = g_{\mu\nu}^{(0)}dx_\mu dx_\nu = d\tau^2 - a_{\mathrm{h}}^2(\tau)d\Omega_k^2$$

Solving Einstein's equations in the bulk one finds

$$a^2 = a_{\mathrm{h}}^2 \left[\left(1 - \frac{\mathcal{H}_{\mathrm{h}}^2 z^2}{4} \right)^2 + \frac{1}{4} \frac{\mu z^4}{a_{\mathrm{h}}^4} \right], \qquad n = \frac{\dot{a}}{\dot{a}_{\mathrm{h}}},$$

where $\mathcal{H}_{\mathrm{h}}^2 = H_{\mathrm{h}}^2 + \frac{\kappa}{a_{\mathrm{h}}^2}$ $\quad H_{\mathrm{h}} = \frac{\dot{a}_{\mathrm{h}}}{a_{\mathrm{h}}}$ Hubble rate at the boundary

P.S. Apostolopoulos, G. Siopsis and N. Tetradis, Phys. Rev. Lett. **102**, (2009)

Comparing the exact solution with the expansion

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + z^2 g_{\mu\nu}^{(2)} + z^4 g_{\mu\nu}^{(4)} + \cdots$$

we can extract $g_{\mu\nu}^{(2)}$ and $g_{\mu\nu}^{(4)}$. Then, using the de Haro et al expression for T^{CFT} we obtain

$$\left\langle T_{\mu\nu}^{\mathrm{CFT}} \right\rangle = t_{\mu\nu} + \frac{1}{4} \left\langle T^{\mathrm{CFT}\,\alpha}{}_{\alpha} \right\rangle g_{\mu\nu}^{(0)}$$

The second term is the conformal anomaly

$$\left\langle T^{\mathrm{CFT}\,\alpha}{}_{\alpha} \right\rangle = \frac{3\ell^3}{16\pi G_5} \frac{\ddot{a}_{\mathrm{h}}}{a_{\mathrm{h}}} \mathcal{H}_{\mathrm{h}}^2$$

The first term is a traceless tensor with non-zero components

$$t_{00} = -3t_i^i = \frac{3\ell^3}{64\pi G_5} \left(\mathcal{H}_{\mathrm{h}}^4 + \frac{4\mu}{a_{\mathrm{h}}^4} - \frac{\ddot{a}_{\mathrm{h}}}{a_{\mathrm{h}}} \mathcal{H}_{\mathrm{h}}^2 \right)$$

Hence, apart from the conformal anomaly, the CFT dual to the time dependent asymptotically AdS_5 bulk metric is a conformal fluid with the equation of state $p_{\mathrm{CFT}} = \rho_{\mathrm{CFT}}/3$
where $\rho_{\mathrm{CFT}} = t_{00}$, $p_{\mathrm{CFT}} = -t_i^i$

from Einstein's equations on the boundary we obtain the holographic Friedmann equation

$$\mathcal{H}_{\mathrm{h}}^2 = \frac{8\pi G_{\mathrm{N}}}{3}\rho_{\mathrm{h}} + \boxed{\frac{\ell^2}{4}\left(\mathcal{H}_{\mathrm{h}}^4 + \frac{4\mu\ell}{a_{\mathrm{h}}^4}\right)}$$

quadratic deviation

dark radiation

Kiritsis, JCAP **0510** (2005) ; Apostolopoulos et al, Phys. Rev. Lett. **102**, (2009)

The second Friedmann equation can be derived from energy-momentum conservation

$$\frac{\ddot{a}_{\mathrm{h}}}{a_{\mathrm{h}}}\left(1-\frac{\ell^2}{2}\mathcal{H}_{\mathrm{h}}^4\right) + \mathcal{H}_{\mathrm{h}}^2 = \frac{4\pi G_{\mathrm{N}}}{3}(\rho_{\mathrm{h}} - 3p_{\mathrm{h}})$$

quadratic deviation

where $\rho_{\mathrm{h}} = T_{00}^{\mathrm{matt}}, \quad p_{\mathrm{h}} = -T^{\mathrm{matt}\,i}{}_{i}$

Holographic map

The time dependent bulk spacetime with metric

$$ds^2_{(5)} == \frac{\ell^2}{z^2}\left[n^2(\tau,z)d\tau^2 - a^2(\tau,z)d\Omega_k^2 - dz^2\right]$$

may be regarded as a *z*-foliation of the bulk with FRW cosmology on each *z*-slice. In particular:

at $z=z_{\rm br}$: RSII cosmology, at $z=0$: holographic cosmology.

A map between *z*-cosmology and *z*=0-cosmology can be constructed using

$$a^2 = a_{\rm h}^2\left[\left(1-\frac{\mathcal{H}_{\rm h}^2 z^2}{4}\right)^2 + \frac{1}{4}\frac{\mu z^4}{a_{\rm h}^4}\right], \qquad n = \frac{\dot a}{\dot a_{\rm h}},$$

and the inverse relation

$$a_{\rm h}^2 = \frac{a}{2}\left(1+\frac{\mathcal{H}^2 z^2}{2} + \mathcal{E}\sqrt{1+\mathcal{H}^2 z^2 - \frac{\mu z^4}{a^4}}\right) \qquad \mathcal{E} = \begin{cases} \pm 1 & \text{one-sided} \\ -1 & \text{two-sided} \end{cases}$$

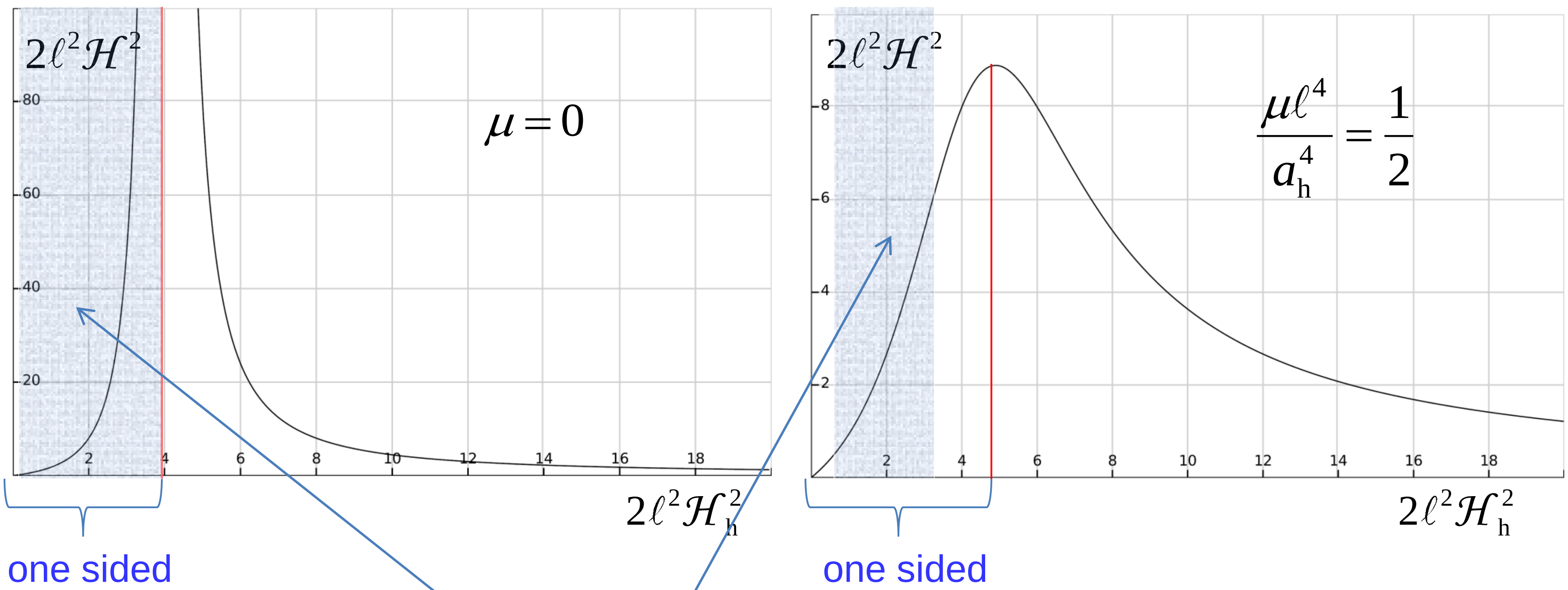
Holographic map

holographic
cosmology

$$
\begin{tikzcd}
z = 0 & \boxed{ds_{\mathrm{h}}^2 = d\tau^2 - a_{\mathrm{h}}^2 d\Omega_k^2} \arrow[r, "\tau \to \tilde{\tau}"] \arrow[d, "z"'] & \boxed{ds_{\mathrm{h}}^2 = \frac{1}{n^2} d\tilde{\tau}^2 - a_{\mathrm{h}}^2 d\Omega_k^2} \arrow[d, "z"] \\
z = z_{\mathrm{br}} & \boxed{ds^2 = n^2 d\tau^2 - a^2 d\Omega_k^2} \arrow[r, "\tau \to \tilde{\tau}"] & \boxed{ds^2 = d\tilde{\tau}^2 - a^2 d\Omega_k^2}
\end{tikzcd}
$$

RSII
cosmology

Relationship between Hubble rates $\mathcal{H}$ at $z=\sqrt{2}\ell$ and $\mathcal{H}_{\mathrm{h}}$ at $z=0$



physical region ($\rho_{\mathrm{h}} \geq 0$)

$$2-2\sqrt{1-\mu\ell^4/a_{\mathrm{h}}^4} \leq \mathcal{H}_{\mathrm{h}}^2\ell^2 \leq 2+2\sqrt{1-\mu\ell^4/a_{\mathrm{h}}^4}$$

The regime of large $\mathcal{H}_{\mathrm{h}}$ violates the weak energy condition $\rho \geq 0$

6. Effective energy density

We analyze two cosmologic scenarios:

Holographic scenario: Primary cosmology is on the AdS boundary at $z = 0$. Observers on the RSII brane on an arbitrary z-slice experience an emergent cosmology which is a reflection of the boundary cosmology.

RSII scenario: Primary cosmology is on the RSII brane at $z = z_{br}$ The cosmology on the $z = 0$ brane emerges as a reflection of the RSII cosmology.

We shall assume the presence of matter on the primary brane only and no matter in the bulk

RSII scenario

In the RSII scenario the primary braneworld is the RSII brane at $z = z_{br}$. Observers at the boundary brane at $z = 0$ experience the emergent cosmology. For simplicity we take $z=\ell$ and we fine tune the tension

$$\sigma = \sigma_0 \equiv 3/(8\pi G_N \ell^2)$$

Then, assuming the modified Friedmann equations hold on the holographic brane, the effective energy density is given by

$$\frac{\rho_h}{\sigma_0} = \frac{4\mathcal{E}(\rho/\sigma_0 + 1 - \mathcal{E})}{(\rho/\sigma_0 + 1 + \mathcal{E})^2 + \mu\ell^4/a^4} \qquad \mathcal{E} = \begin{cases} \pm 1 & \text{one-sided} \\ -1 & \text{two-sided} \end{cases}$$

Thus, the two-sided model with positive energy density and positive μ maps into a holographic cosmology with negative effective energy density ρ_h. For $\mu = 0$ the density ρ_h diverges with ρ as $1/\rho$.

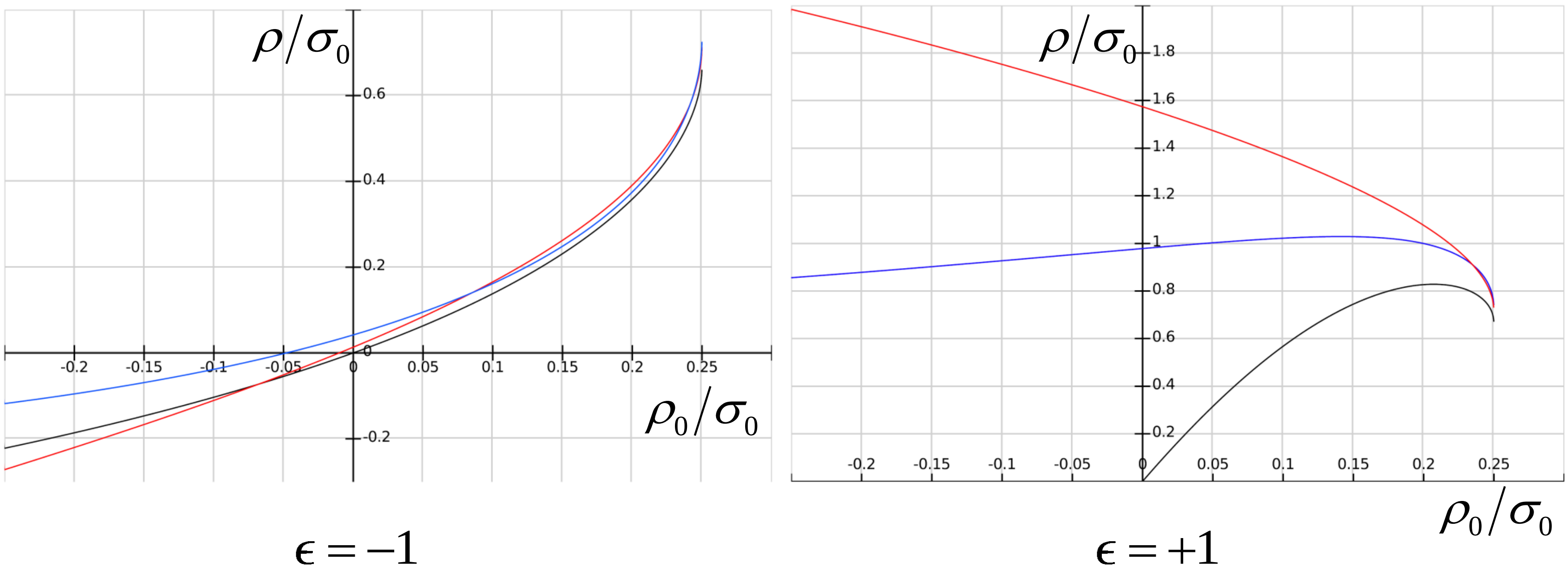
The one-sided model maps into two branches: $\varepsilon = -1$ branch identical with the two-sided map and the $\varepsilon = +1$ branch with smooth positive function $\rho_h = \rho_h(\rho)$.

Holographic scenario

Suppose the cosmology on the $z=0$ brane is known, i.e., the density ρ_h, pressure p_h, and scale a_h are known. If there is no matter in the bulk the induced cosmology on an arbitrary z-slice is completely determined. The general expression for the effective energy density on the RSII brane is rather complicated but simplifies considerably for $z_{br} = \ell$

$$\frac{\rho}{\sigma_0} = \left| \frac{1 + \rho_{\rm h}/\sigma_0 - \epsilon\sqrt{1 - 2\rho_{\rm h}/\sigma_0 - \mu\ell^4/a_{\rm h}^4}}{1 - \rho_{\rm h}/\sigma_0 - \epsilon\sqrt{1 - 2\rho_{\rm h}/\sigma_0 - \mu\ell^4/a_{\rm h}^4}} \right| - \frac{\sigma}{\sigma_0}$$

where $\epsilon = \pm 1$, so the mapping is not unique. The effective energy density ρ diverges in the limit $\rho_h \to 0$.



Effective energy density ρ at z/ℓ = 0.5 (red), 1 (black), 2 (blue line) as a function of ρ_0

For an arbitrary $z_{br} \neq \ell$ in the low density regime (relevant for the one sided version only) $\rho_h \ll \sigma_0, \quad \mu\ell^4 / a_h^4 \ll 1$

a) for $\epsilon = -1$ we find at linear order in μ and quadratic order in ρ_h

$$\frac{\rho}{\sigma_0} = 1 - \frac{\sigma}{\sigma_0} + \frac{z_{\rm br}^2}{\ell^2}\frac{\rho_{\rm h}}{\sigma_0} + \frac{1}{2}\frac{z_{\rm br}^2}{\ell^2}\left(\frac{z_{\rm br}^2}{\ell^2} + 1\right)\frac{\rho_{\rm h}^2}{\sigma_0^2} - \frac{1}{2}\frac{z_{\rm br}^2}{\ell^2}\left(\frac{z_{\rm br}^2}{\ell^2} - 1\right)\frac{\mu\ell^4}{a_{\rm h}^4} + \ldots .$$

and pressure at linear order

$$p = -(\sigma_0 - \sigma) + \frac{z_{\rm br}^2}{\ell^2} p_{\rm h} + \ldots .$$

Hence, at linear order the effective fluid on the RSII brane satisfies the same equation of state as the fluid on the holographic brane. The cosmological constant term will vanish on both branes if the RSII fine tuning condition is imposed.

b) for $\epsilon = +1$ we find

$$\frac{\rho}{\sigma_0} = \overbrace{\frac{z_{\rm br}^2/\ell^2 + 1}{z_{\rm br}^2/\ell^2 - 1} - \frac{\sigma}{\sigma_0}}^{\text{Cosmological constant term}} + \frac{z_{\rm br}^2/\ell^2}{(z_{\rm br}^2/\ell^2 - 1)^2}\frac{\rho_{\rm h}}{\sigma_0} - \frac{z_{\rm br}^2/\ell^2}{2(z_{\rm br}^2/\ell^2 - 1)^3}\frac{\mu\ell^4}{a_{\rm h}^4} + \cdots$$

- The effective density ρ on the RSII brane differs from ρ_h on the holographic brane by a multiplicative constant.
- For $\sigma=\sigma_0$ the effective cosmological constant on the RSII brane does not vanish and is equal to

$$\Lambda_{\rm br} = \frac{6}{\ell^2}\frac{z_{\rm br}^2/\ell^2 + 1}{z_{\rm br}^2/\ell^2 - 1} - \frac{6}{\ell^2}.$$

- The effective density ρ diverges in the limit $z_{br}/\ell \to 1$

Conclusions and outlook

- We have explicitly constructed the mapping between two cosmological braneworlds: holographic and RSII .
- The cosmologies are governed by the corresponding modified Friedman equations.
- There is a clear distinction between 1-sided and 2-sided holographic map with respective 1-sided and 2-sided versions of RSII model.
- In the 2-sided map the low-density regime on the two-sided RSII brane corresponds to the high **negative** energy density on the holographic brane
- The low density regime is maintained on both branes only in the one-sided RSII

Speculations

It is conceivable that we live in a braneworld with emergent cosmology. That is, dark energy and dark matter could be emergent phenomena induced by what happens on the primary braneworld.

For example, suppose our universe is a one-sided RSII braneworld the cosmology of which is emergent in parallel with the primary holographic cosmology. If ρ_0 describes matter with the equation of state satisfying $3p_0 + \rho_0 > 0$, as for, e.g., CDM, we will have an asymptotically de Sitter universe on the RSII brane. If we choose ℓ so that Λ fits the observed value, the quadratic term will be comparable with the linear term today but will strongly dominate in the past and hence will spoil the standard cosmology. However, the standard ΛCDM cosmology could be recovered by including a negative cosmological constant term in ρ_0 and fine tune it to cancel Λ up to a small phenomenologically acceptable contribution.

Thank you