

Scale hierarchies and string cosmology

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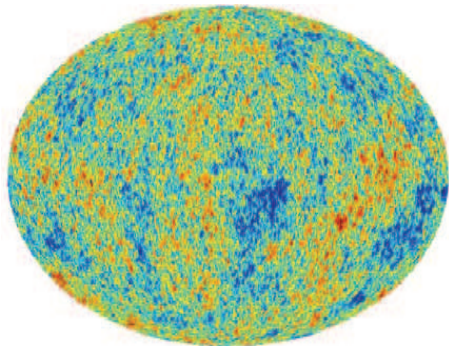


Main predictions → inspirations for BSM physics

- Spacetime supersymmetry but arbitrary breaking scale
- Extra dimensions of space six or seven in M-theory
- Brane-world description of our Universe
matter and gauge interactions may be localised in less dimensions
- Landscape of vacua
- ...

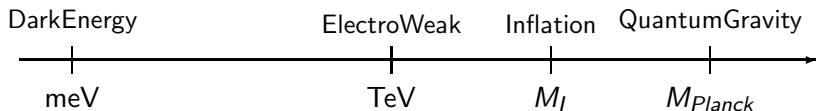
Connect string theory to the real world

- Is it a tool for strong coupling dynamics or a theory of fundamental forces?
- If theory of Nature can it describe both particle physics and cosmology?



Problem of scales

- describe high energy (SUSY?) extension of the Standard Model
unification of all fundamental interactions
 - incorporate Dark Energy
simplest case: infinitesimal (tuneable) +ve cosmological constant
 - describe possible accelerated expanding phase of our universe
models of inflation (approximate de Sitter)
- ⇒ 3 very different scales besides M_{Planck} :



Relativistic dark energy 70-75% of the observable universe

negative pressure: $p = -\rho \Rightarrow$ cosmological constant

$$R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab} = \frac{8\pi G}{c^4} T_{ab} \Rightarrow \rho_\Lambda = \frac{c^4 \Lambda}{8\pi G} = -p_\Lambda$$

Two length scales:

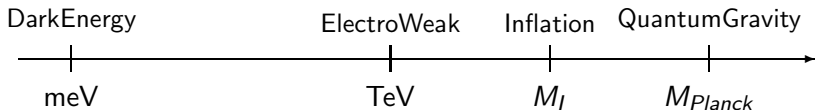
- $[\Lambda] = L^{-2} \leftarrow$ size of the observable Universe

$$\Lambda_{obs} \simeq 0.74 \times 3H_0^2/c^2 \simeq 1.4 \times (10^{26} \text{ m})^{-2}$$

Hubble parameter $\simeq 73 \text{ km s}^{-1} \text{ Mpc}^{-1}$

- $[\frac{\Lambda}{G} \times \frac{c^3}{h}] = L^{-4} \leftarrow$ dark energy length $\simeq 85 \mu\text{m}$

Problem of scales



① they are independent

② possible connections

- M_I could be near the EW scale, such as in Higgs inflation

but large non minimal coupling to explain

- M_{Planck} could be emergent from the EW scale

in models of low-scale gravity and TeV strings

What about M_I ? can it be at the TeV scale?

Can we infer M_I from cosmological data?

I.A.-Patil '14 and '15

- connect inflation and SUSY breaking scales

Inflation in supergravity: main problems

- slow-roll conditions: the eta problem \Rightarrow fine-tuning of the potential

$$\eta = V''/V, \quad V_F = e^K (|DW|^2 - 3|W|^2), \quad DW = W' + K'W$$

K : Kähler potential, W : superpotential

canonically normalised field: $K = X\bar{X} \Rightarrow \eta = 1 + \dots$

- trans-Planckian initial conditions \Rightarrow break validity of EFT
no-scale type models that avoid the η -problem
- stabilisation of the (pseudo) scalar companion of the inflaton
chiral multiplets \Rightarrow complex scalars
- moduli stabilisation, de Sitter vacuum, ...

Starobinsky model of inflation

$$\mathcal{L} = \frac{1}{2}R + \alpha R^2$$

$$\text{Lagrange multiplier } \phi \Rightarrow \mathcal{L} = \frac{1}{2}(1 + 2\phi)R - \frac{1}{4\alpha}\phi^2$$

Weyl rescaling \Rightarrow equivalent to a scalar field with exponential potential:

$$\mathcal{L} = \frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{M^2}{12} \left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2 \quad M^2 = \frac{3}{4\alpha}$$

Note that the two metrics are not the same

supersymmetric extension:

add D-term $\mathcal{R}\bar{\mathcal{R}}$ because F-term \mathcal{R}^2 does not contain R^2

\Rightarrow brings two chiral multiplets

SUSY extension of Starobinsky model

$$K = -3 \ln(T + \bar{T} - C\bar{C}) \quad ; \quad W = MC(T - \frac{1}{2})$$

- T contains the inflaton: $\text{Re } T = e^{\sqrt{\frac{2}{3}}\phi}$
- $C \sim \mathcal{R}$ is unstable during inflation

⇒ add higher order terms to stabilize it

e.g. $C\bar{C} \rightarrow h(C, \bar{C}) = C\bar{C} - \zeta(C\bar{C})^2$ Kallosh-Linde '13

- SUSY is broken during inflation with C the goldstino superfield

→ model independent treatment in the decoupling sgoldstino limit

⇒ minimal SUSY extension that evades stability problem [12]

Non-linear supersymmetry \Rightarrow goldstino mode χ

Volkov-Akulov '73

Effective field theory of SUSY breaking at low energies

Analog of non-linear σ -model \Rightarrow constraint superfields

Rocek-Tseytlin '78, Lindstrom-Rocek '79, Komargodski-Seiberg '09

Goldstino: chiral superfield X_{NL} satisfying $X_{NL}^2 = 0 \Rightarrow$

$$\begin{aligned} X_{NL}(y) &= \frac{\chi^2}{2F} + \sqrt{2}\theta\chi + \theta^2 F & y^\mu &= x^\mu + i\theta\sigma^\mu\bar{\theta} \\ &= F\Theta^2 & \Theta &= \theta + \frac{\chi}{\sqrt{2}F} \end{aligned}$$

$$\mathcal{L}_{NL} = \int d^4\theta X_{NL}\bar{X}_{NL} - \frac{1}{\sqrt{2}\kappa} \left\{ \int d^2\theta X_{NL} + h.c. \right\} = \mathcal{L}_{Volkov-Akulov}$$

R-symmetry with $[\theta]_R = [\chi]_R = 1$ and $[X]_R = 2$

$$F = \frac{1}{\sqrt{2}\kappa} + \dots$$

Non-linear SUSY in supergravity

I.A.-Dudas-Ferrara-Sagnotti '14

$$K = -3 \log(1 - X\bar{X}) \equiv 3X\bar{X} \quad ; \quad W = fX + W_0 \quad \quad X \equiv X_{NL}$$

$$\Rightarrow \quad V = \frac{1}{3}|f|^2 - 3|W_0|^2 \quad ; \quad m_{3/2}^2 = |W_0|^2$$

- V can have any sign contrary to global NL SUSY
- NL SUSY in flat space $\Rightarrow f = 3 m_{3/2} M_p$
- R-symmetry is broken by W_0
- Dual gravitational formulation: $(\mathcal{R} - 6W_0)^2 = 0$ I.A.-Markou '15
↖ chiral curvature superfield
- Minimal SUSY extension of R^2 gravity

Non-linear Starobinsky supergravity [9]

$$K = -3 \ln(T + \bar{T} - X\bar{X}) \quad ; \quad W = MXT + fX + W_0 \quad \Rightarrow$$

$$\mathcal{L} = \frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{M^2}{12} \left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2 - \frac{1}{2}e^{-2\sqrt{\frac{2}{3}}\phi}(\partial a)^2 - \frac{M^2}{18}e^{-2\sqrt{\frac{2}{3}}\phi}a^2$$

- axion a much heavier than ϕ during inflation, decouples:

$$m_\phi = \frac{M}{3}e^{-\sqrt{\frac{2}{3}}\phi_0} \ll m_a = \frac{M}{3}$$

- inflation scale M independent from NL-SUSY breaking scale f

\Rightarrow compatible with low energy SUSY

- however inflaton different from goldstino superpartner

- also initial conditions require trans-planckian values for ϕ ($\phi > 1$) [18]

Inflation from supersymmetry breaking

I.A.-Chatrabhuti-Isono-Knoops '16, '17

Inflaton : goldstino superpartner in the presence of a gauged R-symmetry

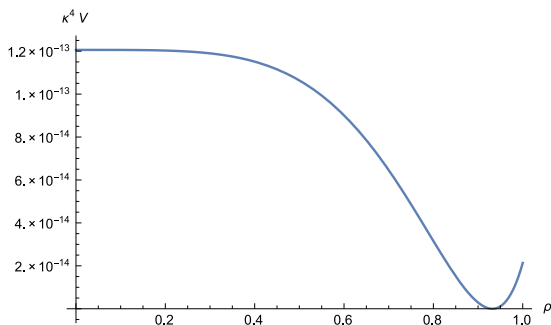
- linear superpotential $W = f X \Rightarrow$ no η -problem

$$\begin{aligned}V_F &= e^K (|DW|^2 - 3|W|^2) \\ &= e^K (|1 + K_X X|^2 - 3|X|^2) |f|^2 \quad K = X\bar{X} \\ &= e^{|X|^2} (1 - |X|^2 + \mathcal{O}(|X|^4)) |f|^2 = \mathcal{O}(|X|^4) \Rightarrow \eta = 0 + \dots\end{aligned}$$

- inflation around a maximum of scalar potential (hill-top) \Rightarrow small field
no large field initial conditions
- gauge R-symmetry: (pseudo) scalar absorbed by the $U(1)_R$
- vacuum energy at the minimum: tuning between V_F and V_D

Two classes of models

- Case 1: R-symmetry is restored during inflation (at the maximum)



- Case 2: R-symmetry is (spontaneously) broken everywhere
(and restored at infinity)

example: toy model of SUSY breaking [18] [27]

Case 1: R-symmetry restored during inflation

$$\mathcal{K}(X, \bar{X}) = \kappa^{-2} X \bar{X} + \kappa^{-4} A (X \bar{X})^2 \quad A > 0$$

$$W(X) = \kappa^{-3} f X \quad \Rightarrow$$

$$f(X) = 1 \quad (+\beta \ln X \text{ to cancel anomalies but } \beta \text{ very small})$$

$$\mathcal{V} = \mathcal{V}_F + \mathcal{V}_D$$

$$\mathcal{V}_F = \kappa^{-4} f^2 e^{X \bar{X} (1 + A X \bar{X})} \left[-3 X \bar{X} + \frac{(1 + X \bar{X} (1 + 2 A X \bar{X}))^2}{1 + 4 A X \bar{X}} \right]$$

$$\mathcal{V}_D = \kappa^{-4} \frac{q^2}{2} [1 + X \bar{X} (1 + 2 A X \bar{X})]^2$$

Assume inflation happens around the maximum $|X| \equiv \rho \simeq 0 \quad \Rightarrow$

Case 1: predictions

slow-roll parameters

$$\eta = \frac{1}{\kappa^2} \left(\frac{V''}{V} \right) = 2 \left(\frac{-4A + x^2}{2 + x^2} \right) + \mathcal{O}(\rho^2) \quad x = q/f$$

$$\epsilon = \frac{1}{2\kappa^2} \left(\frac{V'}{V} \right)^2 = 4 \left(\frac{-4A + x^2}{2 + x^2} \right)^2 \rho^2 + \mathcal{O}(\rho^4) \simeq \eta^2 \rho^2$$

η small: for instance $x \ll 1$ and $A \sim \mathcal{O}(10^{-1})$

inflation starts with an initial condition for $\phi = \phi_*$ near the maximum and ends when $|\eta| = 1$

$$\Rightarrow \text{number of e-folds } N = \int_{end}^{start} \frac{V}{V'} = \kappa \int \frac{1}{\sqrt{2\epsilon}} \simeq \frac{1}{|\eta_*|} \ln \left(\frac{\rho_{end}}{\rho_*} \right)$$

Case 1: predictions

amplitude of density perturbations $A_s = \frac{\kappa^4 V_*}{24\pi^2 \epsilon_*} = \frac{\kappa^2 H_*^2}{8\pi^2 \epsilon_*}$

spectral index $n_s = 1 + 2\eta_* - 6\epsilon_* \simeq 1 + 2\eta_*$

tensor – to – scalar ratio $r = 16\epsilon_*$

Planck '15 data : $\eta \simeq -0.02$, $A_s \simeq 2.2 \times 10^{-9}$, $N \gtrsim 50$

$$\Rightarrow r \lesssim 10^{-4}, H_* \lesssim 10^{12} \text{ GeV}$$

Question: can a 'nearby' minimum exist with a tiny +ve vacuum energy?

Answer: Yes in a 'weaker' sense: perturbative expansion [14] [19]

valid for the Kähler potential but not for the slow-roll parameters

generic V (not fine-tuned) $\Rightarrow 10^{-9} \lesssim r \lesssim 10^{-4}$, $10^{10} \lesssim H_* \lesssim 10^{12} \text{ GeV}$ [33]

impose independent scales: proceed in 2 steps

- 1 SUSY breaking at $m_{SUSY} \sim \text{TeV}$
with an infinitesimal (tuneable) positive cosmological constant

Villadoro-Zwirner '05

I.A.-Knoops, I.A.-Ghilenca-Knoops '14, I.A.-Knoops '15

- 2 Inflation connected or independent? [7] [10] [27]

Toy model for SUSY breaking

Content (besides $N = 1$ SUGRA): one vector V and one chiral multiplet S
with a shift symmetry $S \rightarrow S - icw \leftarrow$ transformation parameter

String theory: compactification modulus or universal dilaton

$$s = 1/g^2 + ia \leftarrow \text{dual to antisymmetric tensor}$$

Kähler potential K : function of $S + \bar{S}$

$$\text{string theory: } K = -p \ln(S + \bar{S})$$

Superpotential: constant or single exponential if R-symmetry $W = ae^{bS}$

$$\int d^2\theta W \text{ invariant}$$

$$b < 0 \Rightarrow \text{non perturbative}$$

can also be described by a generalized linear multiplet [24]

Scalar potential

$$\mathcal{V}_F = a^2 e^{\frac{b}{l} l^{p-2}} \left\{ \frac{1}{p} (pl - b)^2 - 3l^2 \right\} \quad l = 1/(s + \bar{s})$$

Planck units

- $b > 0 \Rightarrow$ SUSY local minimum in AdS space with $l = b/p$
- $b \leq 0 \Rightarrow$ no minimum with $l > 0$ ($p \leq 3$)

but interesting metastable SUSY breaking vacuum when R-symmetry is gauged by V allowing a Fayet-Iliopoulos (FI) term:

$$\mathcal{V}_D = c^2 l (pl - b)^2 \quad \text{for gauge kinetic function } f(S) = S$$

- $b > 0$: $\mathcal{V} = \mathcal{V}_F + \mathcal{V}_D$ SUSY AdS minimum remains
- $b = 0$: SUSY breaking minimum in AdS ($p < 3$)
- $b < 0$: SUSY breaking minimum with tuneable cosmological constant Λ

Scalar potential for $b = 0$

$$V = a^2(p - 3)l^p + c^2 p^2 l^3$$

can be obtained for $p = 2$ and l the string dilaton:

- all geometric moduli fixed by fluxes in a SUSY way
- D-term contribution : D-brane potential (uncancelled tension)
- F-term contribution : tree-level potential (away from criticality)

String realisation : framework of magnetised branes

minimisation and spectrum

Minimisation of the potential: $V' = 0$, $V = \Lambda$

In the limit $\Lambda \approx 0$ ($p = 2$) \Rightarrow [29]

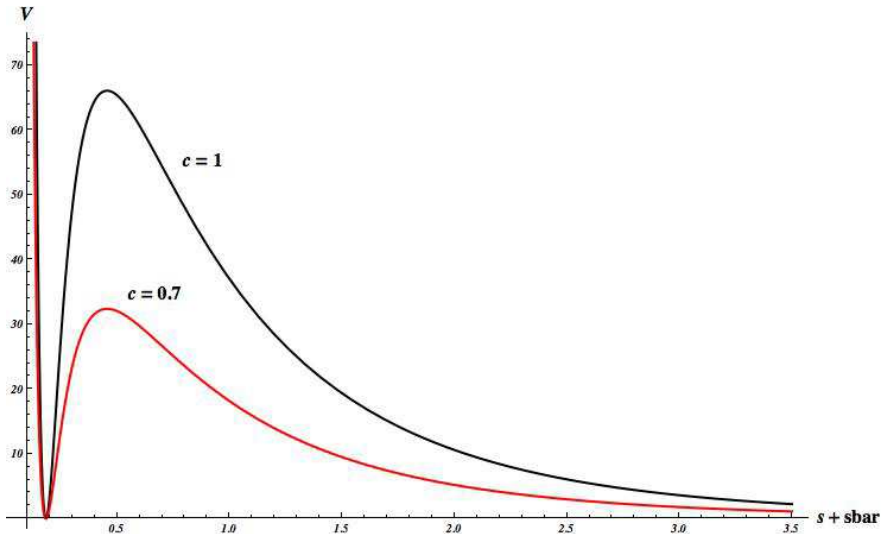
$$b/l = \rho \approx -0.183268 \quad \Rightarrow \langle l \rangle = b/\rho$$

$$\frac{a^2}{bc^2} = 2 \frac{e^{-\rho}}{\rho} \frac{(2-\rho)^2}{2+4\rho-\rho^2} + \mathcal{O}(\Lambda) \approx -50.6602 \quad \Rightarrow c \propto a$$

Physical spectrum:

massive dilaton, $U(1)$ gauge field, Majorana fermion, gravitino

All masses of order $m_{3/2} \approx e^{\rho/2} l a \leftarrow$ TeV scale



[26] [27]

Properties and generalizations

- Metastability of the ground state: extremely long lived

$$l \simeq 0.02 \text{ (GUT value } \alpha_{GUT}/2) m_{3/2} \sim \mathcal{O}(\text{TeV}) \Rightarrow$$

$$\text{decay rate } \Gamma \sim e^{-B} \text{ with } B \approx 10^{300}$$

- Add visible sector (MSSM) preserving the same vacuum

matter fields ϕ neutral under R-symmetry

$$K = -2 \ln(S + \bar{S}) + \phi^\dagger \phi \quad ; \quad W = (a + W_{MSSM}) e^{bS}$$

\Rightarrow soft scalar masses non-tachyonic of order $m_{3/2}$ (gravity mediation)

- Toy model classically equivalent to [19]

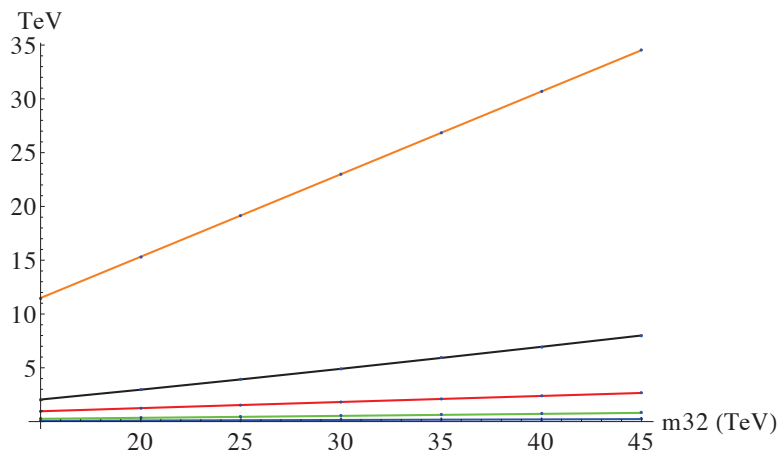
$$K = -p \ln(S + \bar{S}) + b(S + \bar{S}) \quad ; \quad W = a \quad \text{with } V \text{ ordinary } U(1)$$

- Dilaton shift can be identified with $B - L \supset$ matter parity $(-)^{B-L}$

Properties and generalizations

- R-charged fields needed for anomaly cancellation
- A simple (anomaly free) variation: $f = 1$ and $p = 1$
tuning still possible but scalar masses of neutral matter tachyonic
possible solution: add a new field Z in the 'hidden' SUSY sector
 \Rightarrow one extra parameter
- alternatively: add an S -dependent factor in Matter kinetic terms
$$K = -\ln(S + \bar{S}) + (S + \bar{S})^{-\nu} \sum \Phi \bar{\Phi} \quad \text{for } \nu \gtrsim 2.5$$
or the $B - L$ unit charge of SM particles \Rightarrow similar phenomenology
- distinct features from other models of SUSY breaking and mediation
- gaugino masses at the quantum level
 \Rightarrow suppressed compared to scalar masses and A-terms

Typical spectrum



The masses of sbottom squark (yellow), stop (black), gluino (red), lightest chargino (green) and lightest neutralino (blue) as a function of the gravitino mass. The mass of the lightest neutralino varies between ~ 40 and 150 GeV [18] [23]

Case 2 example: toy model of SUSY breaking

I.A.-Chatrabhuti-Isono-Knoops '16

Can the dilaton be the inflaton in the simple model of SUSY breaking based on a gauged shift symmetry?

the only physical scalar left over, partner (partly) of the goldstino
partly because of a D-term auxiliary component

Same potential cannot satisfy the slow roll condition $|\eta| = |V''/V| \ll 1$
with the dilaton rolling towards the Standard Model minimum

\Rightarrow need to create an appropriate plateau around the maximum of V [23]
without destroying the properties of the SM minimum

\Rightarrow study possible corrections to the Kähler potential
only possibility compatible with the gauged shift symmetry

Extensions of the SUSY breaking model

Parametrize the general **correction** to the Kähler potential:

$$K = -p\kappa^{-2} \log \left(s + \bar{s} + \frac{\xi}{b} F(s + \bar{s}) \right) + \kappa^{-2} b(s + \bar{s})$$

$$W = \kappa^{-3} a, \quad f(s) = \gamma + \beta s$$

$$\mathcal{P} = \kappa^{-2} c \left(b - p \frac{1 + \frac{\xi}{b} F'}{s + \bar{s} + \frac{\xi}{b} F} \right)$$

Three types of possible corrections:

- perturbative: $F \sim (s + \bar{s})^{-n}$, $n \geq 0$
- non-perturbative D-brane instantons: $F \sim e^{-\delta(s+\bar{s})}$, $\delta > 0$
- non-perturbative NS5-brane instantons: $F \sim e^{-\delta(s+\bar{s})^2}$, $\delta > 0$

Only the last can lead to slow-roll conditions with sufficient inflation

Slow-roll inflation

$F = \xi e^{\alpha b^2 \phi^2}$ with $\phi = s + \bar{s} = 1/l \Rightarrow$ two extra parameters $\alpha < 0$, ξ
they control the shape of the potential

slow-roll conditions: $\epsilon = 1/2(V'/V)^2 \ll 1$, $|\eta| = |V''/V| \ll 1$

\Rightarrow allowed regions of the parameter space with $|\xi|$ small

additional independent parameters: a, c, b

SM minimum with tuneable cosmological constant Λ : $V' = 0$, $V = \Lambda \approx 0$

$\xi = 0 \Rightarrow b\phi_{min} = \rho_0$, $\frac{a^2}{bc^2} = \lambda_0$ with ρ_0, λ_0 calculable constants [22]

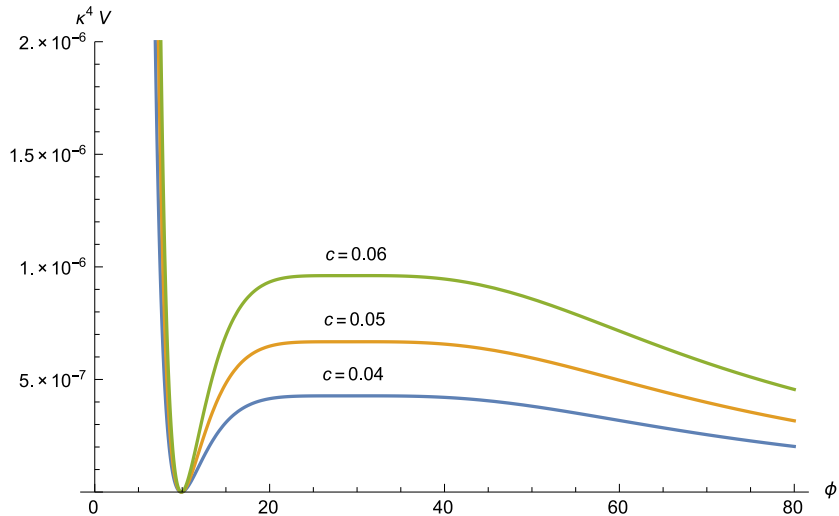
b controls $\phi_{min} \sim 1/g_s$ choose it of order 10

tuning determines a in terms of c overall scale of the potential

$\xi \neq 0 \Rightarrow \rho_0, \lambda_0$ become functions $l(\xi, \alpha), \lambda(\xi, \alpha)$

numerical analysis \Rightarrow mild dependence

$\xi = 0.025, \alpha = -4.8, p = 2, b = -0.018$



Fit Planck '15 data and predictions

$p = 1$: similar analysis \Rightarrow

$$\phi_* = 64.53, \xi = 0.30, \alpha = -0.78, b = -0.023, c = 10^{-13}$$

N	n_s	r	A_s
889	0.959	4×10^{-22}	2.205×10^{-9}

SM minimum: $\langle \phi \rangle \approx 21.53$, $\langle m_{3/2} \rangle = 18.36$ TeV, $\langle M_{A_\mu} \rangle = 36.18$ TeV

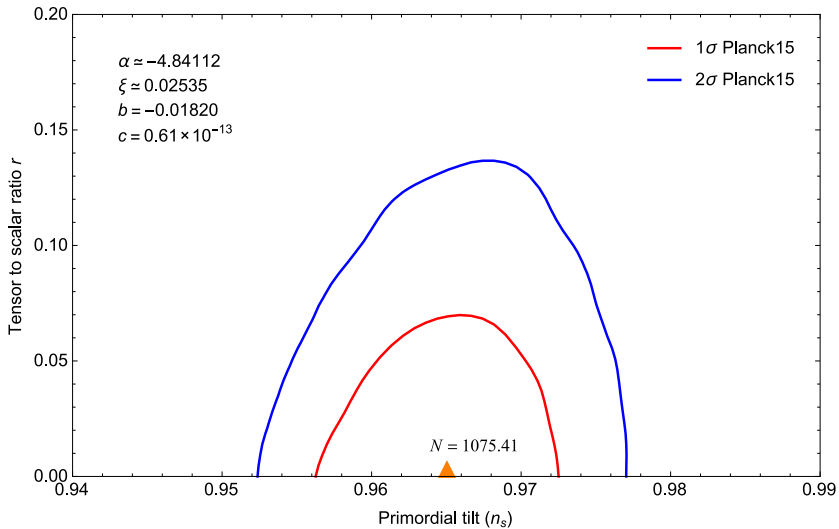
During inflation:

$$H_* = \kappa \sqrt{\mathcal{V}_*/3} = 5.09 \text{ TeV}, m_{3/2}^* = 4.72 \text{ TeV}, M_{A_\mu}^* = 6.78 \text{ TeV}$$

Low energy spectrum essentially the same with $\xi = 0$:

$$m_0^2 = m_{3/2}^2 [-2 + \mathcal{C}], \quad A_0 = m_{3/2} \mathcal{C}, \quad B_0 = A_0 - m_{3/2}$$

$$\mathcal{C} = 1.53 \text{ vs at } \xi = 0: \mathcal{C}_0 = 1.52, m_{3/2}^0 = 17.27, \text{ although } \langle \phi \rangle_0 \approx 9.96 \text{ [14]}$$



Conclusions

String pheno: consistent framework for particle physics and cosmology

Challenge of scales: at least three very different (besides M_{Planck})

electroweak, dark energy, inflation, SUSY?

their origins may be connected or independent

SUSY with infinitesimal (tuneable) +ve cosmological constant

- interesting framework for model building incorporating dark energy
- identify inflaton with goldstino superpartner
inflation at the SUSY breaking scale (TeV?)

General class of models with inflation from SUSY breaking:

(gauged) R-symmetry restored (case 1) or broken (case 2) during inflation

small field, avoids the η -problem, no (pseudo) scalar companion