Tachyon scalar field in DBI and RSII cosmological context *

Dragoljub D. Dimitrijevic†
Faculty of Science and Mathematics, University of Nis, Serbia

Neven Bilic‡
Rudjer Boskovic Institute, Zagreb, Coratia

Goran S. Djordjevic§
Faculty of Science and Mathematics, University of Nis, Serbia

Milan Milosevic¶
Faculty of Science and Mathematics, University of Nis, Serbia

Abstract

The role tachyon fields may play in evolution of early universe is discussed. In particular, we discuss the inflationary scenario based on the tachyon field coupled with the radion of the second Randall-Sundrum model (RSII) and obtained results for the values of observable cosmological parameters, the tensor-to-scalar ratio r and the scalar spectral index n_s.

1. Introduction

The inflationary universe scenario in which the early universe undergoes a rapid expansion provides elegant mechanism to explain and solve horizon, flatness, etc. problems of the standard big-bang cosmology [1, 2, 3]. In a such inflationary universe primordial curvature perturbations, which seed cosmic microwave background temperature anisotropies and the structure formation are generated from vacuum fluctuations of the scalar field, inflaton.

General single field Lagrangian action of the inflaton field

$$S = \int d^4x\sqrt{-g}\mathcal{L}(X(\partial\phi), \phi), \tag{1}$$

* This work has been supported by the Serbian Ministry of Education, Science and Technological Development, Projects OI 174020 and OI 176021, as well as the SEENET-MTP Network under the ICTP grant NT-03.

† e-mail address: ddrag@pmf.ni.ac.rs
‡ e-mail address: bilic@irb.hr
§ e-mail address: gorandj@junis.ni.ac.rs
¶ e-mail address: mmilan@seenet-mtp.info

can be considered, where $X = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ [4, 5]. For example, Lagrangian density of the standard form is

$$\mathcal{L}_{st}(\phi, \partial\phi) = X(\partial\phi) - V(\phi), \tag{2}$$

while popular class comprise tachyon scalar field models [6, 7, 8, 9, 10]

$$\mathcal{L}_{tach}(T, X) = -V(T)\sqrt{1 - 2X(\partial T)}. \tag{3}$$

These models are of particular interest as in these models inflation is driven by the tachyon field originating in string theory [11].

In cosmology, scalar fields can be connected with a perfect fluid which describes (dominant) matter in the Universe. Total action consists of term which describes gravity (Ricci scalar) plus term that describes cosmological fluid (scalar field Lagrangian)

$$S = \int d^4x\sqrt{-g}\left(-\frac{R}{16\pi G} + \mathcal{L}(X, T)\right), \tag{4}$$

from which Einstein equations are obtained

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{5}$$

From the expression of components of the energy-momentum tensor

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = (P + \rho)u_\mu u_\nu - Pg_{\mu\nu}, \tag{6}$$

pressure, matter density and velocity 4-vector can be defined, respectively:

$$P(X, T) \equiv \mathcal{L}(X, T), \quad \rho(X, T) \equiv 2X\frac{\partial\mathcal{L}}{\partial X} - \mathcal{L}(X, T), \quad u_\mu \equiv \frac{\partial_\mu T}{\sqrt{2X}}. \tag{7}$$

Of course, this is model with scalar field minimally coupled to gravity.

2. Tachyon inflation

In this sectin we will focus on tachyon scalar field Lagrangian density

$$\mathcal{L}_{tach}(T, X) = -V(T)\sqrt{1 - g^{\mu\nu}\partial_\mu T\partial_\nu T}, \tag{8}$$

with the equation of motion

$$\left(g^{\mu\nu} - \frac{\partial^\mu T\partial^\nu T}{1 - g^{\rho\sigma}\partial_\rho T\partial_\sigma T}\right)\partial_\mu T\partial_\nu T = -\frac{1}{V(T)}\frac{dV}{dT}(1 - g^{\mu\nu}\partial_\mu T\partial_\nu T). \tag{9}$$

In the case of spatially homogenous scalar field the Friedmann equations are

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\frac{V}{\left(1-\dot{T}^2\right)^{1/2}}, \tag{10}$$

$$\frac{\ddot{T}}{1-\dot{T}^2} + 3H\dot{T} + \frac{V'}{V} = 0. \tag{11}$$

The slow-roll parameters are

$$\epsilon_{i+1} \equiv \frac{d\ln|\epsilon_i|}{dN}, \quad i \geq 0, \quad \epsilon_0 \equiv \frac{H_*}{H}, \tag{12}$$

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_2 = \frac{1}{H}\frac{\ddot{H}}{\dot{H}} + 2\epsilon_1, \tag{13}$$

where H_* is the Hubble parameter at some chosen time [9]. The number of e-folds is

$$N(t) = \int_{t_i}^{t_e} H(t)dt, \tag{14}$$

t_i and t_e correspond to beginning and end of inflation, respectively. In terms of the tachyon field, the slow-roll parameters can be expressed as

$$\epsilon_1 = \frac{3}{2}\dot{T}^2, \quad \epsilon_2 = \sqrt{\frac{2}{3\epsilon_1}}\frac{\epsilon_1'}{H} = 2\frac{\ddot{T}}{H\dot{T}}, \tag{15}$$

where $'$ denotes a derivative with respect to T [12, 13]. The tensor-to-scalar ratio r and the scalar spectral index n_s are defined as [9]

$$r = 16\epsilon_1, \quad n_s = 1 - 2\epsilon_1 - \epsilon_2, \tag{16}$$

and it is understood that ϵ_1 and ϵ_2 take their values at or close to the beginning of inflation. A simple tachyon model can be analyzed in the framework of the second Randall-Sundrum (RSII) model, which will be considered in the next section.

3. Tachyon inflation in an AdS braneworld

Randall-Sundrum model(s) imagine that the real world is a higher-dimensional universe described by warped geometry [14, 15, 16]. More concretely, our universe is a five-dimensional anti-de Sitter space and the elementary particles except for the graviton are localized on a (3 + 1)-dimensional brane(s). A simple cosmological model of this kind is based on the RSII model.

Cosmology on the brane is obtained by allowing the brane to move in the bulk. Equivalently, the brane is kept fixed at fifth coordinate $z = 0$

while making the metric in the bulk time dependent. The fluctuation of the interbrane distance implies the existence of a massless scalar field that causes a distortion of the bulk geometry, called radion.

The bulk spacetime of the extended RSII model

$$ds^2_{(5)} = G_{ab} dX^a dX^b, \tag{17}$$

in Fefferman-Graham coordinates are described by the metric [17]

$$ds^2_{(5)} = \frac{1}{k^2 z^2}\left[\left(1 + k^2 z^2 \eta(x)\right) g_{\mu\nu} dx^\mu dx^\nu - \frac{dz^2}{(1 + k^2 z^2 \eta(x))^2}\right], \tag{18}$$

where $k = 1/l$ is inverse of the AdS curvature radius l and $\eta(x)$ is radion field. Now, if we add dynamical 3-brane, i.e. tachyon field (in terms of induced metric), the action, after integrating out fifth coordinate z becomes [10]:

$$\begin{aligned} S &= \int d^4x \sqrt{-g}\left(-\frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \Phi_{,\mu} \Phi_{,\nu}\right) - \\ &- \int d^4x \sqrt{-g} \frac{\sigma}{k^4 \Theta^4} (1 + k^2 \Theta^2 \eta)^2 \sqrt{1 - \frac{g^{\mu\nu} \Theta_{,\mu} \Theta_{,\nu}}{(1 + k^2 \Theta^2 \eta)^3}}, \end{aligned} \tag{19}$$

where Φ is now canonical radion field, $\eta = \sinh^2\left(\sqrt{4/3\pi G}\Phi\right)$ and Θ is tachyon field. Note that in the absence of radion we end up with tachyon condenzate:

$$S^{(0)}_{br} = -\int d^4x \sqrt{-g} \frac{\lambda}{\Theta^4} \sqrt{1 - g^{\mu\nu} \Theta_{,\mu} \Theta_{,\nu}}. \tag{20}$$

Going back, Lagrangian density (for brane-radion system) we are playing with is

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \Phi_{,\mu} \Phi_{,\nu} - \frac{\lambda \psi^2}{\Theta^4} \sqrt{1 - \frac{g^{\mu\nu} \Theta_{,\mu} \Theta_{,\nu}}{\psi^3}}, \tag{21}$$

where $\psi = 1 + k^2 \Theta^2 \eta$ and $\lambda = \frac{\sigma}{k^4}$.

We will assume the spatially flat FRW spacetime on the observer brane

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2). \tag{22}$$

Going from Lagrangian to Hamiltonian density $\mathcal{H}$

$$\mathcal{H} = \frac{1}{2}\Pi^2_\Phi + \frac{\lambda \psi^2}{\Theta^4} \sqrt{1 + \Pi^2_\Theta \Theta^8 / (\lambda^2 \psi)}, \tag{23}$$

Hamilton's equations become [18]

$$\dot{\Phi} = \frac{\partial \mathcal{H}}{\partial \Pi_\Phi}, \tag{24}$$

$$\dot{\Theta} = \frac{\partial \mathcal{H}}{\partial \Pi_\Theta}, \tag{25}$$

$$\dot{\Pi}_\Phi + 3H\Pi_\Phi = -\frac{\partial \mathcal{H}}{\partial \Phi}, \tag{26}$$

$$\dot{\Pi}_\Theta + 3H\Pi_\Theta = -\frac{\partial \mathcal{H}}{\partial \Theta}, \tag{27}$$

where $\Pi^\mu_\Phi = \frac{\partial \mathcal{L}}{\partial \Phi_{,\mu}}$, $\Pi^\mu_\Theta = \frac{\partial \mathcal{L}}{\partial \Theta_{,\mu}}$, and

$$\Pi_\Phi = \sqrt{g_{\mu\nu}\Pi^\mu_\Phi \Pi^\nu_\Phi}, \qquad \Pi_\Theta = \sqrt{g_{\mu\nu}\Pi^\mu_\Theta \Pi^\nu_\Theta}. \tag{28}$$

In the spatially flat Randall-Sundrum cosmology the Hubble expansion rate H is related to the Hamiltonian via the modified Friedmann equations [19, 20]

$$H \equiv \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}\mathcal{H}\left(1 + \frac{2\pi G}{3k^2}\mathcal{H}\right)}, \tag{29}$$

$$\dot{H} = -4\pi G(\mathcal{H} + \mathcal{L})\left(1 + \frac{4\pi G}{3k^2}\mathcal{H}\right). \tag{30}$$

To solve this system of equations we rescale the time as $t = \tau/k$, expressing the system in terms of dimensionless quantities

$$h = H/k, \quad \phi = \Phi/(k\sqrt{\lambda}), \quad \pi_\phi = \Pi_\Phi/(k^2\sqrt{\lambda})), \tag{31}$$

$$\theta = k\Theta, \quad \pi_\theta = \Pi_\Theta/(k^4\lambda), \tag{32}$$

and obtain dimensionless pressure and energy density

$$\bar{p} \equiv \frac{\mathcal{L}}{k^4\lambda} = \frac{1}{2}\pi_\phi^2 - \frac{\psi^2}{\theta^4}\frac{1}{\sqrt{1 + \theta^8\pi_\theta^2/\psi}}, \tag{33}$$

$$\bar{\rho} \equiv \frac{\mathcal{H}}{k^4\lambda} = \frac{1}{2}\pi_\phi^2 + \frac{\psi^2}{\theta^4}\sqrt{1 + \theta^8\pi_\theta^2/\psi}. \tag{34}$$

Using $\kappa^2 = 8\pi\lambda G k^2$ [9], the Hamilton's dimensionless equations become

$$\dot{\phi} = \pi_\phi, \tag{35}$$

$$\dot{\theta} = \frac{\theta^4\psi\pi_\theta}{\sqrt{1 + \theta^8\pi_\theta^2/\psi}}, \tag{36}$$

$$\dot{\pi}_\phi = -3h\pi_\phi - \frac{\psi}{2\theta^2}\frac{4+3\theta^8\pi_\theta^2/\psi}{\sqrt{1+\theta^8\pi_\theta^2/\psi}}\eta', \tag{37}$$

$$\dot{\pi}_\theta = -3h\pi_\theta + \frac{\psi}{\theta^5}\frac{4-3\theta^{10}\eta\pi_\theta^2/\psi}{\sqrt{1+\theta^8\pi_\theta^2/\psi}}, \tag{38}$$

where the overdot now denotes a derivative with respect to τ and

$$h \equiv \frac{\dot{a}}{a} = \sqrt{\frac{\kappa^2}{3}\bar{\rho}\left(1+\frac{\kappa^2}{12}\bar{\rho}\right)}, \tag{39}$$

$$\psi = 1 + \theta^2\eta, \tag{40}$$

$$\eta = \sinh^2\left(\sqrt{\frac{\kappa^2}{6}}\phi\right), \tag{41}$$

$$\eta' = \frac{d\eta}{d\phi} = \sqrt{\frac{\kappa^2}{6}}\sinh\left(\sqrt{\frac{2\kappa^2}{3}}\phi\right). \tag{42}$$

We can add to this system two additional equations

$$\dot{h} = -\frac{\kappa^2}{2}(\bar{\rho}+\bar{p})\left(1+\frac{\kappa^2}{6}\bar{\rho}\right), \tag{43}$$

$$\dot{N} = h. \tag{44}$$

The first two slow-roll parameters are now

$$\epsilon_1 = -\frac{\dot{h}}{h^2}, \quad \epsilon_2 = 2\epsilon_1 + \frac{\ddot{h}}{h\dot{h}}. \tag{45}$$

The effect of the radion and the tachyon can be seen if we compare the slow-roll parameters for the full model with those for the model with inverse quartic tachyon potential. The number of e-folds is defined through Eq. (44) as

$$N = \int_{\tau_\mathrm{i}}^{\tau_\mathrm{f}} d\tau h. \tag{46}$$

Observable quantities, such as the tensor-to-scalar ratio r and the scalar spectral index n_s are related to the slow-roll parameters

$$r = \frac{\mathcal{P}_\mathrm{T}}{\mathcal{P}_\mathrm{S}}, \tag{47}$$

$$n_{\rm s} = \frac{d \ln \mathcal{P}_{\rm S}}{d \ln k}, \tag{48}$$

where $\mathcal{P}_{\rm S}$ and $\mathcal{P}_{\rm T}$ are the power spectra of scalar and tensor perturbations, respectively (evaluated at the horizon, i.e., for a wave-number satisfying $k = aH$). In this way (at the lowest order in slow-roll parameters)

$$\mathcal{P}_{\rm T} \simeq [1 - 2(1 + C)\epsilon_1] \frac{16GH^2}{\pi}, \tag{49}$$

$$\mathcal{P}_{\rm S} \simeq [1 - 2(1 + C - \alpha)\epsilon_1 - C\epsilon_2] \frac{GH^2}{\pi\epsilon_1}, \tag{50}$$

where $C = -2 + \ln 2 + \gamma \simeq -0.72$, α is a parameter related to the speed of sound expanded in ϵ_1

$$c_{\rm s} = 1 - 2\alpha\epsilon_1 + O(\epsilon_1^2), \tag{51}$$

and is equal to 1/12 [10] (for standard tachyon inflation $\alpha = 1/6$ [9]). To solve the system of equations (35)-(38) we need to choose initial conditions relevant for inflation.

4. Initial conditions

To find appropriate initial value of the field θ, we need to consider the pure tachyon model, with the slow-roll conditions [9]

$$\dot{\theta} \simeq \theta^4 \pi_\theta \ll 1, \quad \dot{\pi}_\theta \ll 3h\pi_\theta. \tag{52}$$

The slow-roll parameters can be approximated as [10]

$$\epsilon_1 \simeq \frac{8\theta^2}{\kappa^2} \left(1 + \frac{\kappa^2}{6\theta^4}\right) \left(1 + \frac{\kappa^2}{12\theta^4}\right)^{-2}, \tag{53}$$

$$\epsilon_2 \simeq \frac{8\theta^2}{\kappa^2} \left(1 + \frac{\kappa^2}{12\theta^4}\right)^{-2} \left[1 + \frac{\kappa^2}{4\theta^4} - \frac{\kappa^2}{6\theta^4} \left(1 + \frac{\kappa^2}{6\theta^4}\right)^{-1}\right]. \tag{54}$$

In this regime $\kappa^2/\theta^4 \gg 1$ and corrections due to the RSII modification bring the equations in the following form

$$h \simeq \frac{1}{6} \frac{\kappa^2}{\theta^4}, \qquad \dot{\theta} \simeq 8\frac{\theta^3}{\kappa^2}, \qquad \ddot{\theta} \simeq 24\frac{\theta^2}{\kappa^2}\dot{\theta}, \tag{55}$$

$$\epsilon_1 \simeq 3\dot{\theta}^2 \simeq 192\frac{\theta^6}{\kappa^4}, \qquad \epsilon_2 \simeq 288\frac{\theta^6}{\kappa^4} \simeq \frac{3}{2}\epsilon_1. \tag{56}$$

So, in the slow-roll regime the tachyon inflation in the RSII modified cosmology proceeds in a quite distinct way compared with that in the standard

FRW cosmology. However, close to and at the end of inflation $\kappa^2/\theta_{\rm f}^4 \ll 1$ and we can safely neglect the RSII cosmology corrections, obtaining (for details see [10])

$$\epsilon_1(\theta_{\rm f}) \simeq \epsilon_2(\theta_{\rm f}) \simeq \frac{8\theta_{\rm f}^2}{\kappa^2} \simeq 1, \tag{57}$$

,

$$h(\theta_{\rm f}) \simeq \frac{8}{\sqrt{3}\kappa}, \tag{58}$$

$$N \simeq \frac{2}{3}\frac{1}{\epsilon_1(\theta_0)} - 1, \tag{59}$$

where θ_0 is initial value of the θ–field deep inside inflation (at the beginning of inflation). Note that in the standard tachyon inflation (for the potential $V = \lambda/\theta^4$) we would obtain for the number of e-folds

$$N_{\rm st.tach} \simeq \frac{\kappa^2}{8\theta_0^2} - 1 \simeq \frac{1}{\epsilon_1(\theta_0)} - 1. \tag{60}$$

The choice of initial values of the radion field is based on the natural scale dictated by observations, i.e., the scale between H and $G^{-1/2}$, and it would be of the order of k or a few orders larger, say in the range 10 to $1000k$. But dimensionless radion field ϕ is rescaled with respect to Φ/k by a factor of $1/\sqrt{\lambda} \simeq 10^{-4}$ so we can choose the initial value $\phi_0 = \phi(0)$ in the range from 0.001 to 0.5.

5. Values of couplings

It is necessary to find a phenomenologically acceptable choice of the couplings λ and κ. The evolution equations do not depend on λ, but its approximate value is needed for choosing appropriate initial conditions for the radion field. The value of λ may be estimated using the observational constraint on the amplitude of scalar perturbations. In this way, the condition

$$H\sqrt{G} \leq \sqrt{\pi A_s} \simeq 8.31 \times 10^{-5}, \tag{61}$$

must be satisfied close to and at the end of inflation, where $A_s \simeq 2.2 \times 10^{-9}$ is power spectrum amplitude measured by Planck [21], yielding

$$\lambda \geq 10^8. \tag{62}$$

Regarding estimation of coupling κ, the tension of a Dp-brane is given by [22]

$$\sigma = \frac{1}{(2\pi)^p \alpha'^{(p+1)/2} g_{\rm s}}, \tag{63}$$

where g_s is the string coupling constant and $1/(2\pi\alpha')$ is the string tension. Using this (and $p = 3$, see [10]) we find a constraint

$$g_s \leq 10^{-12}\frac{M_s}{k}, \tag{64}$$

where $M_s = 1/\sqrt{\alpha'}$. Now, we can choose k and M_s such that the scale hierarchy $H < k < M_s < G^{-1/2}$ is satisfied. It follows that

$$\kappa \geq 8/\sqrt{3}. \tag{65}$$

6. Observational parameters

We are now able to obtain expression for r and n_s. Using (49) and (50) we find up to the second order [10]

$$r = 16\epsilon_1\left[1 - \frac{1}{6}\epsilon_1 + C\epsilon_2\right], \tag{66}$$

$$n_s = 1 - 2\epsilon_1 - \epsilon_2 - \left[2\epsilon_1^2 + \left(2C + \frac{17}{6}\right)\epsilon_1\epsilon_2 + C\epsilon_2\epsilon_3\right], \tag{67}$$

where it is understood that ϵ_1 and ϵ_2 take their values at or close to the beginning of inflation. Using these equations at linear order ($\epsilon_2 \simeq 3\epsilon_1/2$) we can find the approximate relations with the help of (59)

$$n_s \simeq 1 - \frac{7}{3}\frac{1}{N+1}, \tag{68}$$

$$r \simeq \frac{32}{7}(1 - n_s). \tag{69}$$

7. Numerical results

The system of equations (35)-(38) is evolved numerically starting from $\tau = 0$ up to some large τ of the order of 100. The initial values θ_0 and ϕ_0 are chosen as already described. The initial conjugate momenta are taken to be $\pi_{\theta 0} = \pi_{\phi 0} = 0$. The function $N(\tau)$ is solved simultaneously using (44) with $N(0) = 0$. The time evolution of the slow roll parameters ϵ_1 and ϵ_2 are obtained using (45). The inflation ends at a point τ_f at which $\epsilon_1(\tau_f) = 1$. The beginning of inflation at τ_i is then found by requiring

$$N(\tau_f) - N(\tau_i) = N. \tag{70}$$

More details can be found in [10]. Here, we only present in Fig. 1 the $n_s - r$ diagram with 10000 points superimposed on the observational constraints taken from the Planck Collaboration 2015 [21]. Each point in the diagram

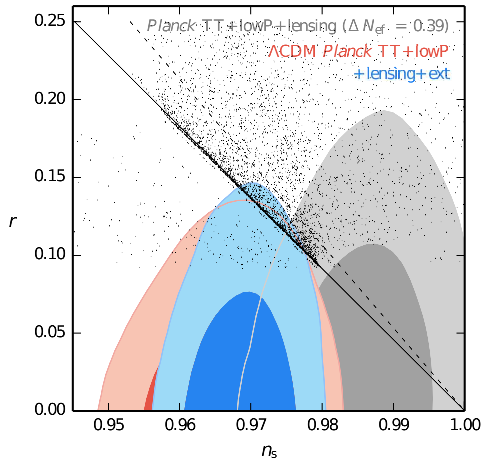

Figure 1: r versus n_{s} diagram with observational constraints from Ref. [21]. The dots represent the calculation in the tachyon-radion model for various N, κ and ϕ_0 chosen randomly in the range $60 \leq N \leq 120$, $1 \leq \kappa \leq 12$ and $0 \leq \phi_0 \leq 0.5$. The full line represents the slow-roll approximation (Eq. (69)) of the RSII model with no radion. The dashed line represents the slow-roll approximation of the standard tachyon model with inverse quartic potential.

corresponds to a set (N, κ, ϕ_0) chosen randomly in the range $60 \leq N \leq 120$, $1 \leq \kappa \leq 12$ and $0 \leq \phi_0 \leq 0.5$.

A still better agreement between our model and the observational data constraints is obtained for a very narrow range of parameters. The best fit is obtained for $\kappa = 1.25$, $\phi_0 = 0.05$ and $115 \leq N \leq 120$ [10].

8. Conclusions

We have investigated a model of inflation based on the dynamics of a $D3$-brane in the AdS5 bulk of the RSII model. The bulk metric is extended to include the back reaction of the radion excitations. The n_s/r relation here is substantially different from the standard one and is closer to the best observational value.

In general, the model is based on the brane dynamics which results in a definite potential with one free parameter only. We have analyzed the simplest tachyon model. In principle, the same mechanism could lead to a more general tachyon potential if the AdS_5 background metric is deformed by the presence of matter in the bulk. The agreement with observations is not ideal and it is fair to say that the present model is disfavored but not excluded.

In [10], we presented results in comparison to the $n_s - r$ diagram, Fig. 1. Here, Fig. 2 and Fig. 3 present histograms for the probability distribution for each parameter individually and compare them with the Planck results. P/P_{max} is the number of parameter values in a given interval divided by the maximal number of parameter values (normalized to unity). Solid lines represent the distribution of Planck's results, while a bar plots represent results for our model.

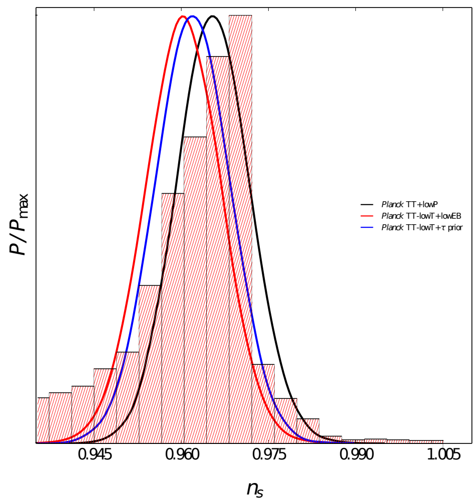

Figure 2: Histogram for density distribution for n_s.

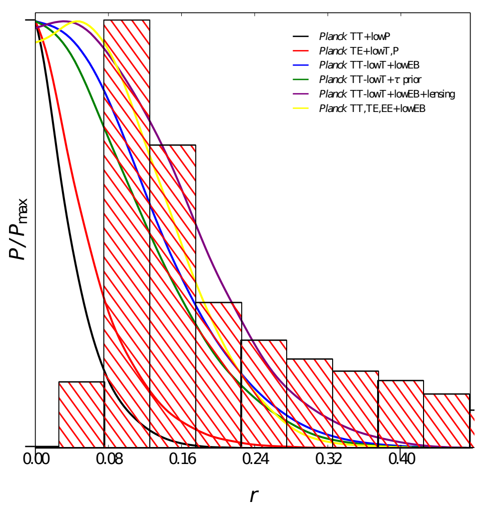

Figure 3: Histogram for density distribution for r.

This is in some sense a step backwards as far as the analysis of results are concerned, but it is easier to understand and analyze the statistics of the observational parameters separately rather then simultaneously. It should be clear in this way how to connect with the probability distribution for the Planck results that we used.

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